Math 217 - October 25, 2010
Solutions

- Systems, matrices, eigenvalues and eigenvectors

1. Find the eigenvalues and eigenvectors for the matrix A.

   What is the general solution to $X' = AX$?

   $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   Solution:

   $$X = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

2. Find the eigenvalues and eigenvectors for the matrix A.

   What is the general solution to $X' = AX$?

   $A = \begin{bmatrix} 1 & 9 \\ 4 & 1 \end{bmatrix}$

   Solution:

   $$X = c_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{7t}$$

3. Find the eigenvalues and eigenvectors for the matrix A.

   What is the general solution to $X' = AX$?

   $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   Solution:

   $$X = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{t}$$

4. Find the eigenvalues and eigenvectors for the matrix A.

   What is the general solution to $X' = AX$?

   $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   Solution:

   $$c_2 \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$$

Lecture Problems

5. Complex eigenvalues and eigenvectors. Find the eigenvalues and eigenvectors of

$A = \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}$

Solution:

$$\lambda_1 = 3 - 2i \qquad v_1 = \begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

$$\lambda_2 = 3 + 2i \qquad v_2 = \begin{bmatrix} 1 \\ -2i \end{bmatrix}$$

6. What is the general the solution to $X' = AX$ where A is in the previous problem?

7. Solve $X' = AX$ where $A = \begin{bmatrix} 1 & -1 \\ 9 & 1 \end{bmatrix}$.

Solution:

$$\lambda_1 = 1 - 3i \qquad v_1 = \begin{bmatrix} 1 \\ 3i \end{bmatrix}$$

$$\lambda_2 = 1 + 3i \qquad v_2 = \begin{bmatrix} 1 \\ -3i \end{bmatrix}$$