Math 217 - October 21, 2010
Solutions

- Matrices:
  - Addition, subraction, multiplication (mult is not communitive but is distributive)
  - Identity matrix, I. I is the matrix such that for any matrix A, we have $AI = IA = A$.
  - Inverse matrix (not all matrices have an inverse). A matrix A has an inverse, usually denoted A^{-1}, if $AA^{-1} = A^{-1}A = I$.
  - Elementary row operations (useful for solving systems of equations and more)
  - Eigenvalue and eigenvectors (square matrices only)

1. The general solution to the differential equation is below. Find the specific solution using the initial condition.

$$X' = \begin{bmatrix} 5 & 2 & 4 \\ -3 & 6 & 2 \\ 3 & -3 & 1 \end{bmatrix} X, \qquad X(0) = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$X = c_1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} e^{7t} + c_2 \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} e^{2t} + c_3 \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} e^{3t}$$

Solution:

$$X = 4 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} e^{7t} - 2 \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} e^{2t} + 3 \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} e^{3t}$$

2. (Fun) If A is a 3×2 matrix and B is a 2×3 matrix such that

$$AB = \begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix}$$

What is BA?

Lecture Problems

3. Find the eigenvalues for the matrices

   (a) $A = \begin{bmatrix} 13 & 5 \\ 2 & 4 \end{bmatrix}$ **Solution:**

$$\lambda_1 = 3, \qquad v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\lambda_2 = 14, \qquad v_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

(b) $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$ **Solution:**

$$\lambda_1 = 3, \qquad v_1 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\lambda_2 = -2, \qquad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(c) $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ **Solution:**

$$\lambda_1 = 1 - 2i, \qquad v_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$\lambda_2 = 1 + 2i, \qquad v_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

(d) $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ **Solution:**

$$\lambda_1 = 3, \qquad v_1 = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

$$\lambda_2 = -4, \qquad v_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$\lambda_3 = 0, \qquad v_3 = \begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix}$$

4. Find the eigenvectors associated to each of the eigenvalues in the previous problem.