

Math 217 - October 21, 2010
Solutions

• Systems

1.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$
$$C = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$$

(a) $AB = \begin{bmatrix} 6 & 20 & 21 \\ 4 & 12 & 17 \end{bmatrix}$

(b) $BA = \text{Error!}$

(c) $BC = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $CB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $A(3D) = \begin{bmatrix} 21 \\ -21 \end{bmatrix}$

(f) $BD = \begin{bmatrix} 11 \\ 15 \\ 10 \end{bmatrix}$

(g) $DB = \text{Error!}$

(h) $A(B + C) = \begin{pmatrix} 8 & 19 & 21 \\ -8 & 15 & 25 \end{pmatrix}$

(i) $AB + AC = \begin{pmatrix} 8 & 19 & 21 \\ -8 & 15 & 25 \end{pmatrix}$

(j) $(A(B + C))D = \begin{bmatrix} 71 \\ 19 \end{bmatrix}$

(k) $ABD + BCD = \begin{bmatrix} 71 \\ 19 \end{bmatrix}$

2. Write the system below as a matrix equation: $X' = P(t)X + f(t)$.

$$x_1' = 2x_1 + tx_2 + t^2$$

$$x_2' = 5x_1 - 7x_2$$

Solution:

$$X' = \begin{bmatrix} 2 & t \\ 5 & -7 \end{bmatrix} X + \begin{bmatrix} t^2 \\ 0 \end{bmatrix}$$

3. Write the system below as a matrix equation: $X' = P(t)X + f(t)$.

$$x_1' = -4x_2 - x_3 - \sin^2 t$$

$$x_2' = 7x_1 - 8x_2 - t$$

$$x_3' = \pi x_2 - \pi^2 x_3$$

Solution:

$$X' = \begin{bmatrix} 0 & -4 & -1 \\ 7 & -8 & 0 \\ 0 & \pi & -\pi^2 \end{bmatrix} X + \begin{bmatrix} -\sin^2 t \\ -t \\ 0 \end{bmatrix}$$

4. (Just for fun) The matrix below has the properties: all entries are nonnegative, there are at most 2 positive entries per row or column, the sum of every row and column is 3.

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

How many different 4×4 matrices are there with these properties?

(How many different $n \times n$ matrices are there with these properties?)

Lecture Problems

Linear system outline:

- (a) Is the system linear? If so, convert to 1st order linear system and write as a matrix system: $X' = P(t)X + F(t)$.
 - (b) Superposition true for homogeneous equations.
 - (c) Use Wronskian to test for linear independence of solutions.
 - (d) Homogeneous Equations: General solution will be a linear combination of n linearly independent solutions.
 - (e) Nonhomogeneous equations: general solution will be a particular solution plus the complementary solution.
 - (f) Initial value problem: Use initial values to solve for constants.
5. Verify the given solutions are solutions and then, using the Wronskian, determine if the solutions are linearly independent.

$$X' = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} X, \quad X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{7t}, \quad X_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix} e^{-t}$$