Math 217 - October 21, 2010

- Systems

1.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$$

(a) $AB =$

(b) $BA =$

(c) $BC =$

(d) $CB =$

(e) $A(3D) =$

(f) $BD =$

(g) $DB =$

(h) $A(B + C) =$

(i) $AB + AC =$

(j) $(A(B + C))D =$

(k) $ABD + BCD =$

2. Write the system below as a matrix equation: $X' = P(t)X + f(t)$.

$$\begin{aligned} x_1' &= 2x_1 + tx_2 + t^2 \\ x_2' &= 5x_1 - 7x_2 \end{aligned}$$

3. Write the system below as a matrix equation: $X' = P(t)X + f(t)$.

$$\begin{aligned} x_1' &= -4x_2 - x_3 - \sin^2 t \\ x_2' &= 7x_1 - 8x_2 - t \\ x_3' &= \pi x_2 - \pi^2 x_3 \end{aligned}$$

4. (Just for fun) The matrix below has the properties: all entries are nonnegative, there are at most 2 positive entries per row or column, the sum of every row and column is 3.

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

How many different 4×4 matrices are there with these properties?

(How many different $n \times n$ matrices are there with these properties?)

Lecture Problems

Linear system outline:

(a) Is the system linear? If so, convert to 1st order linear system and write as a matrix system: $X' = P(t)X + F(t)$.

(b) Superposition true for homogeneous equations.

(c) Use Wronskian to test for linear independence of solutions.

(d) Homogeneous Equations: General solution will be a linear combination of n linearly independent solutions.

(e) Nonhomogeneous equations: general solution will be a particular solution plus the complementary solution.

(f) Initial value problem: Use initial values to solve for constants.

5. Verify the given solutions are solutions and then, using the Wronskian, determine if the solutions are linearly independent.

$$X' = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} X, \qquad X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{7t}, \quad X_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix} e^{-t}$$