Math 217 - October 19, 2010
Solutions

- Exam 2: 3.1-3.8
- Linear equations, theory: Linearly independent functions, superposition, existence and uniqueness, Wronskian, complementary solution, particular solution.
- Techniques: characteristic equation, complex roots and Euler's formula, method of undetermed coefficients, method of variation of parameters.
- Applications: mechanical vibrations, over damped, under damped, critically damped, resonance, practical resonance, endpoint problems and eigenvalues.

1. $x'' + 25x = f(t)$. Find $f(t)$ so that there is resonance.

   Solution: Something like $\sin 5t$ or $\cos 5t$.

2. Solve $4y'' + 400y = 0$

   Solution: $y = c_1 \cos 10x + c_2 \sin 10x$.

3. Solve $y'' + 8y' = 16y = 0$

   Solution: $y = c_1 e^{-4x} + c_2 x e^{-4x}$.

4. Solve $y'' - y = 3$

   Solution: $y = c_1 e^{x} + c_2 e^{-x} - 3$

5. Solve $y'' + 2y' - 3y = 8e^{x}$

   Solution: $y = c_1 c^{x} + c_2 e^{-3x} + 2xe^{x}$

6. Solve $y'' + 2y' - 3y = -x^2$

   Solution: $y = c_1 c^{x} + c_2 e^{-3x} + (9x^2 + 12x + 14)/27$.

7. Solve $2y'' + 3y' + y = 3 \sin x$

   Solution: $y = c_1 e^{-x} + c_2 e^{-x/2} - (3 \sin x + 9 \cos x)/10$

8. Solve and determine $\lim_{t\to\infty} y(t)$: $y'' + 4y' + 3y = 0$, $y(0) = 1, y'(0) = 2$.

   Solution: $y = 2e^{-t} - e^{-4t}$.

9. Consider the nonhomogeneous and homogeneous equations $y'' + p(t)y' + q(t)y = f(t)$ and $y'' + p(t)y' + q(t)y = 0$. Let y_1 and y_2 be solution to the nonhomogeneous equation and let y_3 and y_4 be solutions to the homogeneous equations. True or false:

   (a) $y_1 + y_2$ is a solution to the nonhomogeneous equation. (F)

   (b) $y_3 + y_4$ is a solution to the homogeneous equation. (T)

   (c) $y_1 + y_3$ is a solution to the nonhomogeneous equation. (T)

(d) $5y_1$ is a solution to the nonhomogeneous equation. (F)

(e) $5y_3$ is a solution to the homogeneous equation. (T)

10. What is the best correct form for a particular solution to $y'' - 10y' + 25y = 6e^{5t}$.

    Solution: $y_p = At^2e^{5t}$

11. A two pound object stretches a springt 6 inches. That spring is set into motion from equilibrium with a downward velocity of 2 feet per second (no damping, no external force). After how many seconds does the spring first return to equilibrium?

    Solution: $(1/16)x'' + 4x = 0$. Solving gives $x = (1/4)\sin 8t$. Returns to equilibrium after time $t = \pi/8$.

12. Determine if overdamped, critically damped or underdamped: $\frac{1}{2}x'' + 4x' + 6x = 0$

    Solution: Overdamped

13. Solve $y'' + 9y = 18\sec 3t$

    Solution: $y = c_1\cos 3t + c_2\sin 3t - 2\cos 3t\ln|\sec 3t| + 6t\sin 3t$

14. Determine the best form for the particular solution to $y' + y' - 2y = 7t^2e^t$.

    Solution: $y_p = At^3e^t + Bt^2e^t + Cte^t$