

Math 217 - October 18, 2010

Solutions

- Systems

1. Perform the matrix multiplications

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -4 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 6 \\ 0 & -1 & 11 \\ 1 & 0 & -2 \end{bmatrix}$$

2. Perform the matrix multiplications

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix}$$

3. Solve the system

$$\begin{aligned} x' &= -y & x(0) &= 2 \\ y' &= 10x - 7y & y(0) &= -1 \end{aligned}$$

Solution: $x = c_1 e^{-5t} + c_2 e^{-2t}$, $y = -x' = -5c_1 e^{-5t} - 2c_2 e^{-2t}$. Initial conditions give $c_1 = -1, c_2 = 3$.

Lecture Problems

4. Transform the equation into a first order system.

$$x^{(4)} + t^2 x^{(3)} - 5x'' + x = \sin^2 t$$

Solution:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= x_4 \\ x_4' &= -t^2 x_4 + 5x_3 - x_1 + \sin^2 t \end{aligned}$$

5. Transform the system into a first order system

$$\begin{aligned} x'' &= 2x - x' + y + t^2 \\ y'' &= y' - 2x' - t^3 \end{aligned}$$

Solution: Let $x_1 = x, x_2 = x', y_1 = y, y_2 = y'$:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= 2x_1 - x_2 + y_1 + t^2 \\ y_1' &= y_2 \\ y_2' &= y_2 - 2x_2 - t^3 \end{aligned}$$

6. In the previous problem, suppose you had an initial condition of $x(0) = 1, x'(0) = 2, y(0) = 3, y'(0) = 4$. What are the correct initial conditions for your system?

Solution:

$$x_1(0) = 1$$

$$x_2(0) = 2$$

$$y_1(0) = 3$$

$$y_2(0) = 4$$