Math 217 - October 14, 2010

Solutions

- Boundary Value Problems
- Systems

1. For any λ, find all solutions to

$$y'' + \lambda y = 0, \quad y(0) = 0, y(1) = 0$$

Solution: Eigen values: $\lambda_n = n^2\pi^2$, $y_n = \sin n\pi x$.

2. For any λ, find all solutions to

$$y'' + 2y' + \lambda y = 0, \quad y(0) = 0, y(1) = 0$$

Solution: $r^2 + 2r + \lambda = (r+1)^2 + (\lambda - 1)$ so:

(a) If $\lambda < 1$ then there are two real roots and $y = c_1 e^{-r_1 x} + c_2 e^{r_2 x}$. Solving for c_1 and c_2 yeilds $c_1 = c_2 = 0$.

(b) If $\lambda = 1$ then $y = c_1 e^{-x} + c_2 x e^{-x}$. Boundary conditions give $c_1 = c_2 = 0$.

(c) If $\lambda > 1$ let $\lambda = 1 + \alpha^2$ (to simplify the solution). Then $y = e^{-x}(c_1 \cos(\alpha x) + c_2 \sin(\alpha x))$ Boundary conditions give $\lambda_n = n^2\pi^2 + 1$ and $y_n = e^{-x} \sin n\pi x$.

Lecture Problems

3. Solve the system

$$\begin{aligned} x' &= -2y & x(0) &= 1 \\ y' &= 2x & y(0) &= 0 \end{aligned}$$

Solution: $x = \cos 2t, y = \sin 2t$.

4. Solve the system

$$\begin{aligned} x' &= y & x(0) &= 1 \\ y' &= 6x - y & y(0) &= 2 \end{aligned}$$

Solution: $x = e^{-3t}, y = 2e^{2t}$.