Math 217 - October 12, 2010
Solutions

- Forced oscillations

1. Solve. Let $m = 1, k = 25, F_0 = 27, \omega = 4$:

$$x'' + 25x = 27\cos 4t, \quad x(0) = x'(0) = 0$$

Solution: $x_p = 3\cos 4t$, $x = 3\cos 4t - 3\cos 5t$

2. Apply the trig identity $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$ to the previous problem. (Can you use this to plot the solution?)

Solution: $A - B = 4, A + B = 5$ gives $A = 9/2$ and $B = 1/2$. $x = 6\sin(9t/2)\sin(t/2)$.

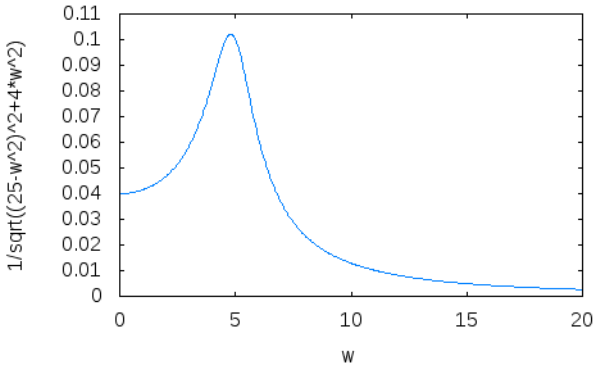
3. Discuss the nature of solutions to the following differential equations

$$\begin{aligned} mx'' + kx &= F_0 \cos \omega t \\ mx'' + cx' + kx &= F_0 \cos \omega t \end{aligned}$$

Lecture Problems

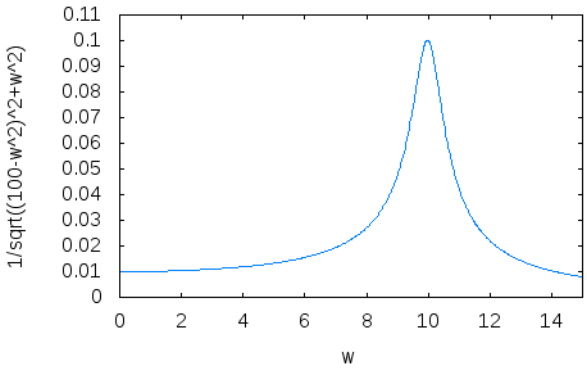
4. Find the practical resonance for the system where $m = 1, c = 2, k = 25$. Use the formula for amplitude given in class and below: $C = \dfrac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$

Solution: You can solve to get $\omega = \sqrt{23}$.



5. Find the practical resonance for the system where $m = 1, c = 1, k = 100$.

Solution: You can solve to get $\omega = \sqrt{199/2}$.



6. Solve the following boundary value problems. The boundary conditions for each are $y(0) = 0$ and $y'(\pi) = 0$

   (a) $y'' - 2y = 0$

   (b) $y'' - y = 0$

   (c) $y'' + y = 0$

   (d) $y'' + 2y = 0$