# Math 217 - October 11, 2010

**Solutions**

- Variation of parameters
- Forced oscillations

**Warning: Problems 1-3 have many typos in them. That said, the actual differential equations in the problems are all doable and you should be able to do these using variation of parameters.**

1. Variation of parameters. Below is an outline of doing variation of parameters. You are to complete the problem. (And, you should double check the work at home.) Start with the equation $y'' + 2y' + y = e^{-x} \ln x$

   (a) $y_c = c_1 e^{-x} + c_2 x e^{-x}$

   (b) $y_c = u_1 e^{x} + u_2 e^{-x}$. Write down the correct equations for $u_1'$ and $u_2'$.
   **Solution:** $u_1' = -x \ln x$, $u_2' = \ln x$

   (c) Using your formulas for $u_1'$ and $u_2'$, you can find that $u_1 = x^2/4(1 - 2\ln x)$ and $u_2 = x(\ln x - x)$. What is $y_p$?
   **Solution:** $y_p = x^2/4(1 - 2\ln x)e^{-x} + x(\ln x - x)xe^{-x}$

   (d) What is the general solution.
   **Solution:** $y = c_1 e^{-x} + c_2 x e^{2x} + x^2/4(1 - 2\ln x)e^{-x} + x(\ln x - x)xe^{-x}$

   (e) If you had initial condition $y(1) = 1$, $y'(1) = 0$, find the particular solution.
   **Solution:** Solve to get $c_1 = -1/4$ and $c_2 = 1 + e$. $y = -(1/4)e^{-x} + (1+e)xe^{2x} + x^2/4(1 - 2\ln x)e^{-x} + x(\ln x - x)xe^{-x}$

2. Variation of parameters. $y'' - 2y' - 3y = 64xe^{-x}$

   (a) $y_c = c_1 e^{-x} + c_2 e^{3x}$

   (b) $y_c = u_1 e^{-x} + u_2 e^{3x}$. Write down the correct equations for $u_1'$ and $u_2'$.
   **Solution:** $u_1' = -16xe^{2x}$, $u_2' = 16xe^{-2x}$

   (c) Using your formulas for $u_1'$ and $u_2'$, you can find that $u_1 = -4(2x-1)e^{2x}$ and $u_2 = -4(2x+1)e^{-2x}$. What is $y_p$?
   **Solution:** $y_p = -4(2x-1)e^{x} - 4(2x+1)e^{x} = -16xe^{x}$

   (d) What is the general solution.
   **Solution:** $y = c_1 e^{-x} + c_2 e^{3x} - 16xe^{x}$

   (e) If you had initial condition $y(1) = 5$, $y'(1) = 2$, find the particular solution.
   **Solution:** Solve to get $c_1 = -3/4$ and $c_2 = 23/4$. $y = -(3/4)e^{-x} + (23/4)e^{3x} - 16xe^{x}$

3. Variation of parameters. $y'' - 3y' + 2y = (1 + e^{-x})^{-1}$

   (a) $y_c = c_1 e^{x} + c_2 e^{2x}$

(b) $y_c = u_1 e^x + u_2 e^{2x}$. Write down the correct equations for $u_1'$ and $u_2'$.

**Solution:** $u_1' = -e^{-x}/(e^{-x}+1)$, $u_2' = e^{-2x}/(e^{-x}+1)$

(c) Using your formulas for $u_1'$ and $u_2'$, you can find that $u_1 = \ln(e^{-x}+1)$ and $u_2 = \ln(e^x+1) - e^{-x} - x$. What is $y_p$?

**Solution:** $y_p = e^x \ln(e^{-x}+1) + e^{2x}\left[\ln(e^x+1) - e^{-x} - x\right]$.

(d) What is the general solution.

**Solution:** $y = c_1 e^x + c_2 e^{2x} + e^x \ln(e^{-x}+1) + e^{2x}\left[\ln(e^x+1) - e^{-x} - x\right]$.

(e) If you had initial condition $y(0) = 0$, $y'(0) = 0$, find the particular solution.

**Solution:** Solve to get $c_1 = -\ln 2$ and $c_2 = 1 - \ln 2$. $y = -(\ln 2)e^x + (1 - \ln 2)e^{2x} + e^x \ln(e^{-x}+1) + e^{2x}\left[\ln(e^x+1) - e^{-x} - x\right]$.

## Lecture Problems

4. Solve. Let $m = 1, k = 25, F_0 = 27, \omega = 4$:

$$x'' + 25x = 27\cos 4t, \quad x(0) = x'(0) = 0$$

**Solution:** $x_p = 3\cos 4t$, $x = 3\cos 4t - 3\cos 5t$

5. Apply the trig identity $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$ to the previous problem. (Can you use this to plot the solution?)

**Solution:** $A - B = 4, A + B = 5$ gives $A = 9/2$ and $B = 1/2$. $x = 6\sin(9t/2)\sin(t/2)$.