Math 217 - November 30, 2010
Solutions

1. Reindex the following series so that the x term is x^n.

   (a)

$$\sum_{n=1}^{\infty} n^2 c_n x^{n+2} = \sum_{n=3}^{\infty} (n-2)^2 c_{n-2} x^n$$

   (b)

$$\sum_{n=1}^{\infty} \frac{n}{\sin n} c_{n+3} x^{n-5} = \sum_{n=-4}^{\infty} \frac{n+5}{\sin(n+5)} c_{n+8} x^n$$

2. "Peel" off the first few terms so that the sum is $\sum_{n=4}^{\infty}$.

$$\sum_{n=3}^{\infty} \frac{nc_{n-3}}{n+5} x^n = \frac{4c_0}{8} x^3 + \sum_{n=4}^{\infty} \frac{nc_{n-3}}{n+5} x^n$$

3. "Peel" off the first few terms so that the sum is $\sum_{n=6}^{\infty}$.

$$\sum_{n=2}^{\infty} \frac{n-5}{n+5} c_n x^{n+2} = \frac{-3}{7} c_2 x^4 + \frac{-2}{8} c_3 x^5 + \frac{-1}{9} c_4 x^6 + \frac{0}{10} c_5 x^7 + \sum_{n=6}^{\infty} \frac{n-5}{n+5} c_n x^{n+2}$$

4. Reindex the recurrence relation so that it is $c_n =$

   (a) $c_{n+2} = \frac{n+1}{4(n+2)} c_n$ becomes $c_n = c_n = \frac{n-1}{4n} c_{n-2}$

   (b) $c_{n+2} = nc_n + c_{n-2}$ becomes $c_n = (n-2)c_{n-2} + c_{n-4}$

   (c) $c_{n+2} = \frac{nc_n + c_{n-2}}{(n+2)(n+1)}$ becomes $c_n = \frac{(n-2)c_{n-2} + c_{n-4}}{n(n-1)}$

5. (As started in class yesterday) Given the recurrance relation, find the first several terms and find the complete the solution.

$$c_{n+2} = -\frac{c_n}{n+2}$$

Lecture Problems

6. Given the recurrance relation, complete the solution.

$$c_{n+2} = -\frac{n+4}{n+2}c_n$$

Solution:

$$\begin{aligned} c_{2n} =& (-1)^n(n+1)c_0 \\ c_{2n+1} =& (-1)^n\frac{1}{3}(2n+3)c_1 \end{aligned}$$

7. Given the recurrance relation, complete the solution.

$$c_{n+2} = \frac{(n-3)(n-4)}{(n+1)(n+2)}c_n$$

Solution:

$$y = c_0(1 + 6x^2 + x^4) + c_1(x + x^3)$$

8. Given the recurrance relation, find the solution to the DE

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = c_n - 2c_{n-1}$$

Solution:

$$\begin{aligned} y =& c_0\left(1 + \frac{1}{2}x^2 - 2x^3 + \frac{1}{2}x^4 - 3x^5 + \frac{9}{2}x^6 - 4x^7 + \frac{21}{2}x^8 + -13x^9 + \cdots\right) \\ &+ c_1\left(x + x^3 - 2x^4 + x^5 - 4x^6 + 5x^7 - 6x^8 + 13x^9 + \cdots\right) \end{aligned}$$

9. Given the recurrance relation, find the solution to the DE

$$c_2 = c_3 = c_4 = c_5 = 0, \quad c_{n+2} = \frac{c_{n-4} + n^2c_{n+1}}{(n+1)(n+2)}$$

Solution:

$$\begin{aligned} y =& c_0\left(1 + \frac{1}{30}x^6 + \frac{5}{252}x^7 + \frac{5}{392}x^8 + \frac{5}{576}x^9 + \cdots\right) \\ &+ c_1\left(x + \frac{1}{42}x^7 + \frac{3}{196}x^8 + \frac{1}{96}x^9 + \frac{1}{135}x^{10} + \cdots\right) \end{aligned}$$