Math 217 - November 29, 2010

- Series Solutions

1. What is an ordinary point?

2. What is a singular point?

3. Why do we care about ordinary points and singular points?

4. If you find a series solution at an ordinary point, what can you say about the radius of convergence for the series solution?

5. $$(1-t^2)y'' - 6ty' - 4y = 0$$

   Start with the series $y = \sum_{n=0}^{\infty} c_n t^n$.

   (a) Find a recurrance relation for the coefficients.

   (b) Use the recurrance relation to find c_0, c_1, c_2, c_3, c_4, c_5, and c_6.

   (c) Try to find the solution to the equation.

6. $$(2x-x^2)y'' - 6(x-1)y' - 4y = 0$$

   Start with the series $y = \sum_{n=0}^{\infty} c_n (x-1)^n$.

   (a) Find a recurrance relation for the coefficients.

   (b) Use the recurrance relation to find c_0, c_1, c_2, c_3, c_4, c_5, and c_6.
   Hint: $2x - x^2 = 1 - (x-1)^2$

   (c) Try to find the solution to the equation.

Lecture Problems

7. Use a substitution to transform the differential equations

   (a)
   $$(x+1)^2 y'' - 2(x+1)y' - y = x, \quad y(-1) = 5, y'(-1) = 2$$

   (b)
   $$(x+1)^2 y'' - 2(x+1)y' - y = x, \quad y(3) = 5, y'(3) = 2$$

8. Given the recurrance relation, complete the solution.

$$c_{n+2} = -\frac{n+4}{n+2}c_n$$

9. Given the recurrance relation, complete the solution.

$$c_{n+2} = \frac{(n-3)(n-4)}{(n+1)(n+2)}c_n$$

10. Given the recurrance relation, find the solution to the DE

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = c_n - 2c_{n-1}$$

11. Given the recurrance relation, find the solution to the DE

$$c_2 = c_3 = c_4 = c_5 = 0, \quad c_{n+2} = \frac{c_{n-4} + n^2 c_{n+2}}{(n+1)(n+2)}$$