Math 217 - November 15, 2010
Solutions

- Delta function review
- Power series
- General overview for exam

$\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$	$\mathscr{L}\left\{t^n\right\} = \dfrac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^a\right\} = \dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{\dfrac{1}{\sqrt{t}}\right\} = \dfrac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}\,dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathscr{L}\left\{\cosh(at)\right\} = \dfrac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \dfrac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \dfrac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \dfrac{a}{s^2+a^2}$
$\mathscr{L}\left\{\dfrac{1}{2a^3}(\sin at - at\cos at)\right\} = \dfrac{s}{(s^2+a^2)^2}$	$\mathscr{L}\left\{\dfrac{t}{2a}\sin at\right\} = \dfrac{s}{(s^2+a^2)}$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{f''(t)\right\} = s^2\,F(s) - sf(0) - f'(0)$
$\mathscr{L}\left\{u_a(t)\right\} = \mathscr{L}\left\{u(t-a)\right\} = \dfrac{1}{s}e^{-sa}$	$\mathscr{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$
$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$	
$\mathscr{L}\left\{\delta(t)\right\} = 1$	$\mathscr{L}\left\{\delta(t-a)\right\} = \mathscr{L}\left\{\delta_a(t)\right\} = e^{-sa}$
$\mathscr{L}\left\{\int_0^t f(u)\,du\right\} = \dfrac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\dfrac{1}{s}F(s)\right\} = \int_0^t f(u)\,du$
$(f*g)(t) = \int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{t\,f(t)\right\} = -F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^n F^{(n)}(s)$
$\mathscr{L}\left\{\dfrac{1}{t}f(t)\right\} = \int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\int_s^\infty F(u)\,du\right\}$
$\cosh t = \dfrac{1}{2}(e^t + e^{-t})$	$\sinh t = \dfrac{1}{2}(e^t - e^{-t})$

1. Convert using a Laplace Transform $x'' + 4x = 6\delta(t), \qquad x(0) = 0, x'(0) = 0$

   Solution: $s^2X + 4X = 6$, $x = 3\sin 2t$.

2. Convert using a Laplace Transform $x'' + 4x = 6\delta(t-3), \qquad x(0) = 0, x'(0) = 0$

   Solution: $s^2X + 4X = 6e^{-3s}$, $x = 3u(t-3)\sin 2(t-3)$.

3. Convert using a Laplace Transform $x'' + 4x = 6\delta(t), \qquad x(0) = 1, x'(0) = 2$

   Solution: $s^2X - s - 2 + 4X = 6$, $x = 4\sin 2t + \cos 2t$.

4. Convert using a Laplace Transform $x'' + 4x = 6\delta(t-3), \qquad x(0) = 1, x'(0) = 2$

   Solution: $s^2X - s - 2 + 4X = 6e^{-3s}$, $X = e^{-3s}\frac{6}{s^2+4} + \frac{s+2}{s^2+4}$ $x = 6u(t-3)\sin 2(t-3) + \sin 2t + \cos 2t$

5. Solve the previous problems.

Lecture Problems

6. Write out the series centered at $x = 0$ for the functions

   (a) $e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$

   (b) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$

   (c) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$

   (d) $\cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!}x^{2n}$

   (e) $\sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}x^{2n+1}$

   (f) $\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}x^n$

   (g) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

   (h) $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots$

7. Chapter 4 (4.1 only)

   (a) What is a system of differential equations?

   (b) What is a linear system of differential equations?

   (c) Why are first order systems of differential equations so important.

   (d) How can you visualize a first order system of differential equations?

   (e) What are some of the applications we were able to solve using systems?

8. Chapter 5 (5.1 and 5.2 only)

   (a) What are some operations you can do with matrices?

   (b) What is the identity matrix?

   (c) What is an inverse matrix?

   (d) How can you write a system of differential equations using matrices?

   (e) How does the Wronsian apply to linear systems?

   (f) What does the general solution look like for a homogeneous linear system?

   (g) What is an eigenvalue and an eigenvector?

   (h) How many eigenvalues and eigenvectors do you expect a matrix to have?

   (i) How do you find eigenvalues and eigenvectors?

   (j) How do eigenvalues and eigenvectors help you solve systems?

   (k) What about complex eigenvalues and eigenvectors?

9. Chapter 7 (7.1-7.6)

(a) What is the definition of the Laplace transform?

(b) What is the Gamma function?

(c) How do we typically find the Laplace transform of a function?

(d) Given a function of s, how can we tell if it is the Laplace transform of some function of t?

(e) Can you take the Laplace transform of any function of t?

(f) How do you Laplace transform a differential equation?

(g) Once you transfor a differential equation, how does this help you solve the differential equation?

(h) What is our main technique for taking the inverse Laplace transform?

(i) What is the convolution of two functions?

(j) How do convolutions help us solve differential equations?

(k) How do you take the transform of a piecewise definied function?

(l) What is the delta function and how do you take its Laplace transform?

(m) What is the unit step function and why do we care about it?