Math 217 - November 11, 2010

Solutions

$\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$	$\mathscr{L}\left\{t^{n}\right\} = \dfrac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^{a}\right\} = \dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{\dfrac{1}{\sqrt{t}}\right\} = \dfrac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \displaystyle\int_0^\infty e^{-t}t^{x-1}\,dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathscr{L}\left\{\cosh(at)\right\} = \dfrac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \dfrac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \dfrac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \dfrac{a}{s^2+a^2}$
$\mathscr{L}\left\{\dfrac{1}{2a^3}(\sin at - at\cos at)\right\} = \dfrac{s}{(s^2+a^2)^2}$	$\mathscr{L}\left\{\dfrac{t}{2a}\sin at\right\} = \dfrac{s}{(s^2+a^2)}$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{f''(t)\right\} = s^2\,F(s) - sf(0) - f'(0)$
$\mathscr{L}\left\{u_a(t)\right\} = \mathscr{L}\left\{u(t-a)\right\} = \dfrac{1}{s}e^{-sa}$	$\mathscr{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$
$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$	
$\mathscr{L}\left\{\delta(t)\right\} = 1$	$\mathscr{L}\left\{\delta(t-a)\right\} = \mathscr{L}\left\{\delta_a(t)\right\} = e^{-sa}$
$\mathscr{L}\left\{\displaystyle\int_0^t f(u)\,du\right\} = \dfrac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\dfrac{1}{s}F(s)\right\} = \displaystyle\int_0^t f(u)\,du$
$(f*g)(t) = \displaystyle\int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{t\,f(t)\right\} = -F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^nF^{(n)}(s)$
$\mathscr{L}\left\{\dfrac{1}{t}f(t)\right\} = \displaystyle\int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\displaystyle\int_s^\infty F(u)\,du\right\}$
$\cosh t = \dfrac{1}{2}(e^t+e^{-t})$	$\sinh t = \dfrac{1}{2}(e^t-e^{-t})$

1. Write the functions by using unit step functions, $u_a(t) = u(t-a)$.

   (a) $f(t) = \begin{cases} \pi & t<4 \\ e & t>4 \end{cases}$

   (b) $f(t) = \begin{cases} e^{2t} & t<4 \\ t^3 & t>4 \end{cases}$

   (c) $f(t) = \begin{cases} \cos t & t<2\pi \\ \sin 2t & 2\pi<t<4\pi \\ \cos 3t & t>4\pi \end{cases}$

Lecture Problems

2. Let $f(t) = \begin{cases} \pi & t<4 \\ e & t>4 \end{cases}$

$$
\begin{aligned}
\mathscr{L}\{f(t)\} =& \mathscr{L}\{\pi(1-u(t-4))+eu(t-4)\}\\
=& \mathscr{L}\{\pi+(e-\pi)u(t-4)\}\\
=& \frac{1}{s}+\frac{(e-\pi)e^{-4s}}{s}
\end{aligned}
$$

3. Let $f(t) = \begin{cases} e^{2t} & t < 4 \\ t^3 & t > 4 \end{cases}$

$$
\begin{aligned}
\mathscr{L}\{f(t)\} =& \mathscr{L}\{e^{2t}(1-u(t-4))+t^3u(t-4)\}\\
=& \mathscr{L}\{e^{2t}+(t^3-e^{2t})u(t-4)\}\\
=& \frac{1}{s-2}+e^{-4s}\mathscr{L}\{(t+4)^3\}-e^{-4s}\mathscr{L}\{e^{2(t+4)}\}
\end{aligned}
$$

4. Let $f(t) = \begin{cases} \cos t & t < 2\pi \\ \sin 2t & 2\pi < t < 4\pi \\ \cos 3t & t > 4\pi \end{cases}$

$$
\begin{aligned}
\mathscr{L}\{f(t)\} =& \mathscr{L}\{(\cos t)(1-u_{2\pi})+(\sin 2t)(u_{2\pi}-u_{4\pi})\\
& \quad + (\cos 3t)(u_{4\pi})\}\\
=& \mathscr{L}\{\cos t+(-\cos t+\sin 2t)u_{2\pi}+\\
& \qquad (-\sin 2t+\cos 3t)u_{4\pi}\}\\
=& \frac{s}{s^2+1}+e^{-2\pi s}\mathscr{L}\{-\cos(t+2\pi)+\sin 2(t+2\pi)\}\\
& \qquad + e^{-4\pi s}(-\sin 2(t+4\pi)+\cos 3(t+4\pi)\}
\end{aligned}
$$

5.

$$
\mathscr{L}^{-1}\left\{\frac{e^{-3s}}{s^3}\right\} = \frac{1}{2}u(t-3)(t-3)^2
$$

6.

$$
\mathscr{L}^{-1}\left\{\frac{1-e^{-2\pi s}}{s^2+4}\right\} = \frac{1}{2}\sin 2t-\frac{1}{2}u(t-2\pi)\sin 2(t-2\pi)
$$