Math 217 - November 8, 2010
Solutions

$\mathscr{L}\left\{e^{at}\right\} = \frac{1}{s-a}$	$\mathscr{L}\left\{t^n\right\} = \frac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^a\right\} = \frac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{u_a(t)\right\} = \frac{e^{-sa}}{s}$
$\mathscr{L}\left\{\cosh(at)\right\} = \frac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \frac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \frac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \frac{a}{s^2+a^2}$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$
$\mathscr{L}\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\frac{1}{s}F(s)\right\} = \int_0^t f(u)\,du$
$(f*g)(t) = \int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{-t\,f(t)\right\} = F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^nF^{(n)}(s)$
$\mathscr{L}\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\int_s^\infty F(u)\,du\right\}$

1.

$$\mathscr{L}^{-1}\left\{\frac{39}{(s+2)(s^2+9)}\right\} = 2\,\sin(3\,t) - 3\,\cos(3\,t) + 3\,e^{-2\,t}$$

2.

$$\mathscr{L}^{-1}\left\{\frac{8s}{s^2+10s+9}\right\} = 9\,e^{-9\,t} - e^{-t}$$

3.

$$\mathscr{L}^{-1}\left\{\frac{8s}{s^2+10s+29}\right\} = e^{-5\,t}\,\left(8\,\cos(2\,t) - 20\,\sin(2\,t)\right)$$

Lecture Problems

4. Use convolutions

   (a) $(t)*(e^t) = e^t - t - 1$

   (b) $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\} = e^t - t - 1$

   (c) $\mathscr{L}^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\} = \dfrac{1}{a}(\sin(at))*(\cos(at)) = \dfrac{1}{2}t\sin(at)$

   (d) $\mathscr{L}^{-1}\left\{\dfrac{1}{(s^2+a^2)^2}\right\} = \dfrac{1}{a^2}(\sin(at))*(\sin(at)) = \dfrac{1}{2a^3}(\sin at - at\cos at)$

5. Use $\mathscr{L}\left\{-tf(t)\right\} = F'(s)$:

(a)

$$\begin{aligned}\mathscr{L}\left\{t\sin at\right\} =&(-1)\frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)\\ =&\frac{2\,a\,s}{\left(s^2+a^2\right)^2}\end{aligned}$$

(b)

$$\begin{aligned}\mathscr{L}\left\{t^2\sin at\right\} =&(-1)^2\frac{d^2}{ds^2}\left(\frac{a}{s^2+a^2}\right)\\ =&\frac{2\,a\,\left(3\,s^2-a^2\right)}{\left(s^2+a^2\right)^3}\end{aligned}$$

(c) Suppose $y(0)=1$.

$$\mathscr{L}\left\{ty'\right\} = -\frac{d}{ds}(sY-1) = Y'$$

(d) Suppose $y(0)=1$ and $y'(0)=2$.

$$\mathscr{L}\left\{ty''\right\} = -\frac{d}{ds}(s^2Y-s-2) = -2sY-s^2Y'-1$$

6. Use $\mathscr{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\,du$:

(a)

$$\mathscr{L}\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{u^2+1}\,du = \frac{\pi}{2}-\tan^{-1}s$$

(b)

$$\begin{aligned}\mathscr{L}^{-1}\left\{\frac{2s}{(s^2+1)^2}\right\} =&t\mathscr{L}^{-1}\left\{\int_s^\infty \frac{2u}{(u^2+1)^2}\,du\right\}\\ =&t\mathscr{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = t\sin t\end{aligned}$$

7. Problems for home

(a) $(t)*(\sin t) = t-\sin t$

(b) $(\cos 2t)*(\sin 3t) = -\frac{3}{5}\cos 3t+\frac{3}{5}\sin 2t$