

Math 217 - November 8, 2010

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$	$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$
$\mathcal{L}\{t^a\} = \frac{\Gamma(a+1)}{s^{a+1}}$	$\mathcal{L}\{u_a(t)\} = \frac{e^{-sa}}{s}$
$\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2-a^2}$	$\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2-a^2}$
$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$	$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$
$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$	$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
$\mathcal{L}\left\{\int_0^t f(u) du\right\} = \frac{1}{s}F(s)$	$\mathcal{L}^{-1}\left\{\frac{1}{s}F(s)\right\} = \int_0^t f(u) du$
$(f * g)(t) = \int_0^t f(u)g(t-u) du$	$\mathcal{L}\{(f * g)(t)\} = F(s)G(s)$
$\mathcal{L}\{-tf(t)\} = F'(s)$	$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$
$\mathcal{L}\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(u) du$	$\mathcal{L}^{-1}\{F(s)\} = t\mathcal{L}^{-1}\left\{\int_s^\infty F(u) du\right\}$

1.

$$\mathcal{L}^{-1} \left\{ \frac{39}{(s+2)(s^2+9)} \right\} =$$

2.

$$\mathcal{L}^{-1} \left\{ \frac{8s}{s^2+10s+9} \right\} =$$

3.

$$\mathcal{L}^{-1} \left\{ \frac{8s}{s^2+10s+29} \right\} =$$

1.

$$\mathcal{L}^{-1} \left\{ \frac{39}{(s+2)(s^2+9)} \right\} = 2 \sin(3t) - 3 \cos(3t) + 3e^{-2t}$$

2.

$$\mathcal{L}^{-1} \left\{ \frac{8s}{s^2+10s+9} \right\} = 9e^{-9t} - e^{-t}$$

3.

$$\mathcal{L}^{-1} \left\{ \frac{8s}{s^2+10s+29} \right\} = e^{-5t} (8 \cos(2t) - 20 \sin(2t))$$

Lecture Problems

4. Use convolutions

(a) $(t) * (e^t) =$

(b) $\mathcal{L}^{-1} \left\{ \frac{1}{s^2(s-1)} \right\} =$

(c) $\mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\} =$

(d) $\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + a^2)^2} \right\} =$

Lecture Problems

4. Use convolutions

$$(a) \quad (t) * (e^t) = e^t - t - 1$$

$$(b) \quad \mathcal{L}^{-1} \left\{ \frac{1}{s^2(s-1)} \right\} = e^t - t - 1$$

$$(c) \quad \mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\} = \frac{1}{a} (\sin(at)) * (\cos(at)) = \frac{1}{2} t \sin(at)$$

$$(d) \quad \mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + a^2)^2} \right\} = \frac{1}{a^2} (\sin(at)) * (\sin(at)) = \\ \frac{1}{2a^3} (\sin at - at \cos at)$$

5. Use $\mathcal{L}\{-tf(t)\} = F'(s)$:

(a)

$$\mathcal{L}\{t \sin at\} =$$

(b)

$$\mathcal{L}\{t^2 \sin at\} =$$

(c) Suppose $y(0) = 1$.

$$\mathcal{L}\{ty'\} =$$

(d) Suppose $y(0) = 1$ and $y'(0) = 2$.

$$\mathcal{L}\{ty''\} =$$

5. Use $\mathcal{L}\{-tf(t)\} = F'(s)$:
(a)

$$\begin{aligned}\mathcal{L}\{t \sin at\} &= (-1) \frac{d}{ds} \left(\frac{a}{s^2 + a^2} \right) \\ &= \frac{2as}{(s^2 + a^2)^2}\end{aligned}$$

(b)

$$\begin{aligned}\mathcal{L}\{t^2 \sin at\} &= (-1)^2 \frac{d^2}{ds^2} \left(\frac{a}{s^2 + a^2} \right) \\ &= \frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3}\end{aligned}$$

(c) Suppose $y(0) = 1$.

$$\mathcal{L}\{ty'\} = -\frac{d}{ds}(sY - 1) = Y'$$

(d) Suppose $y(0) = 1$ and $y'(0) = 2$.

$$\mathcal{L}\{ty''\} = -\frac{d}{ds}(s^2Y - s - 2) = -2sY - s^2Y' - 1$$

6. Use $\mathcal{L} \left\{ \frac{f(t)}{t} \right\} = \int_s^\infty F(u) du$:

(a)

$$\mathcal{L} \left\{ \frac{\sin t}{t} \right\} =$$

(b)

$$\mathcal{L}^{-1} \left\{ \frac{2s}{(s^2 + 1)^2} \right\} =$$

6. Use $\mathcal{L} \left\{ \frac{f(t)}{t} \right\} = \int_s^\infty F(u) du$:

(a)

$$\mathcal{L} \left\{ \frac{\sin t}{t} \right\} = \int_s^\infty \frac{1}{u^2 + 1} du = \frac{\pi}{2} - \tan^{-1} s$$

(b)

$$\begin{aligned} \mathcal{L}^{-1} \left\{ \frac{2s}{(s^2 + 1)^2} \right\} &= t \mathcal{L}^{-1} \left\{ \int_s^\infty \frac{2u}{(u^2 + 1)^2} du \right\} \\ &= t \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 1} \right\} = t \sin t \end{aligned}$$

7. Problems for home

(a) $(t) * (\sin t) =$

(b) $(\cos 2t) * (\sin 3t) =$

7. Problems for home

(a) $(t) * (\sin t) = t - \sin t$

(b) $(\cos 2t) * (\sin 3t) = -\frac{3}{5} \cos 3t + \frac{3}{5} \sin 2t$