

Math 217 - November 1, 2010

Solutions

$$\begin{array}{ll} \mathcal{L}\{e^{at}\} = \frac{1}{s-a} & \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \\ \mathcal{L}\{t^a\} = \frac{\Gamma(a+1)}{s^{a+1}} & \mathcal{L}\{u_a(t)\} = \frac{e^{-sa}}{s} \\ \mathcal{L}\{\cosh(at)\} = \frac{s}{s^2 - a^2} & \mathcal{L}\{\sinh(at)\} = \frac{a}{s^2 - a^2} \\ \mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2} & \mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2} \end{array}$$

1. $\int_a^\infty e^{-st} dt = \frac{e^{-as}}{s}$

2. $\mathcal{L}\{3\sin 5t - 6e^{7t}\} = \frac{25}{s^2+25} - \frac{6}{s-7}$

3. $\mathcal{L}\{t^4 - t^2 + \sqrt{t}\} = \frac{24}{s^5} - \frac{2}{s^3} + \frac{\sqrt{\pi}}{2s^{3/2}}$

4. $\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{1}{24}t^4$

5. $\mathcal{L}^{-1}\left\{\frac{1}{2s^2+32}\right\} = \frac{1}{8}\sin(4t)$

6. $\mathcal{L}^{-1}\{s\} = \text{Error!}$

7. $\mathcal{L}^{-1}\{1\} = \text{Error!}$

Lecture Problems

8. Determine which functions below might have inverse Laplace transforms

(a) $F(s) = \frac{1}{s^{7/3}}$

(b) $F(s) = \frac{s^2-1}{s^2+s+10}$

9. Solve the equation using the Laplace transform

$$y' - 4y = 1, y(0) = 1$$

Solution:

$$\begin{aligned} sY - 4Y - 1 &= \frac{1}{s} \\ Y &= \frac{s+1}{s(s-4)} = -\frac{1}{4} + \frac{5}{4(s-4)} \\ y &= \frac{5e^{4t}}{4} - \frac{1}{4} \end{aligned}$$

10. Solve the equation using the Laplace transform

$$x'' - x' - 6x = \cos 2t, \quad x(0) = 0, x'(0) = 1$$

Solution:

$$s^2 X - sX - 6X - 1 = \frac{s}{s^2 + 4}$$

$$X = \frac{s^2 + s + 4}{(s - 3)(s + 2)(s^2 + 4)}$$
$$= -\frac{5s + 2}{52(s^2 + 4)} - \frac{3}{20(s + 2)} + \frac{16}{65(s - 3)}$$

$$x = -\frac{\sin 2t}{52} - \frac{5 \cos 2t}{52} - \frac{3e^{-2t}}{20} + \frac{16e^{3t}}{65}$$