Name:
ID:

- 18 multiple choice questions (weighted equally) worth 100 points total.
- No graphing calculators! Any non-graphing scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- You are allowed both sides of 3×5 "cheat card" for the exam.

$\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$	$\mathscr{L}\left\{t^n\right\} = \dfrac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^a\right\} = \dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{\dfrac{1}{\sqrt{t}}\right\} = \dfrac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \displaystyle\int_0^\infty e^{-t}t^{x-1}\,dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathscr{L}\left\{\cosh(at)\right\} = \dfrac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \dfrac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \dfrac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \dfrac{a}{s^2+a^2}$
$\mathscr{L}\left\{\dfrac{1}{2a^3}(\sin at - at\cos at)\right\} = \dfrac{1}{(s^2+a^2)^2}$	$\mathscr{L}\left\{\dfrac{t}{2a}\sin at\right\} = \dfrac{s}{(s^2+a^2)^2}$
$\mathscr{L}\left\{\dfrac{1}{2a}(\sin at + at\cos at)\right\} = \dfrac{s^2}{(s^2+a^2)^2}$	$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{f''(t)\right\} = s^2\,F(s) - sf(0) - f'(0)$
$\mathscr{L}\left\{u_a(t)\right\} = \mathscr{L}\left\{u(t-a)\right\} = \dfrac{1}{s}e^{-sa}$	$\mathscr{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$
$\mathscr{L}\left\{\delta(t)\right\} = 1$	$\mathscr{L}\left\{\delta(t-a)\right\} = \mathscr{L}\left\{\delta_a(t)\right\} = e^{-sa}$
$\mathscr{L}\left\{\displaystyle\int_0^t f(u)\,du\right\} = \dfrac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\dfrac{1}{s}F(s)\right\} = \displaystyle\int_0^t f(u)\,du$
$(f*g)(t) = \displaystyle\int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{t\,f(t)\right\} = -F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^n F^{(n)}(s)$
$\mathscr{L}\left\{\dfrac{1}{t}f(t)\right\} = \displaystyle\int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\displaystyle\int_s^\infty F(u)\,du\right\}$
$\cosh t = \dfrac{1}{2}(e^t + e^{-t})$	$\sinh t = \dfrac{1}{2}(e^t - e^{-t})$

1.

$$\frac{dy}{dx} = ye^x, \qquad y(0) = 2e$$

What is $y(1)$ (rounded to 2 decimal places)?

(a) -5.74
(b) 5.43
(c) 5.74
(d) 8.96
(e) 17.43
(f) 28.21
(g) 30.31
(h) 57.03

2.

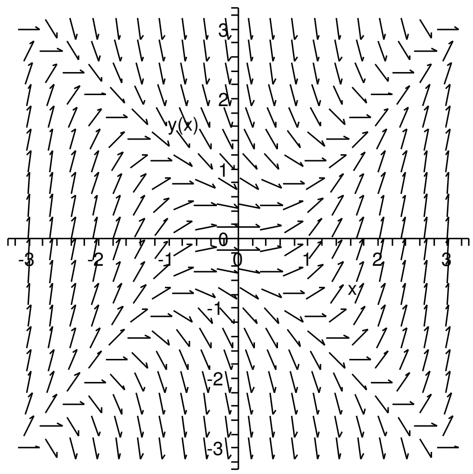
$$xy' - 3y = x^3, \qquad y(1) = 3$$

What is $y(2)$ (rounded to 2 decimal places)?

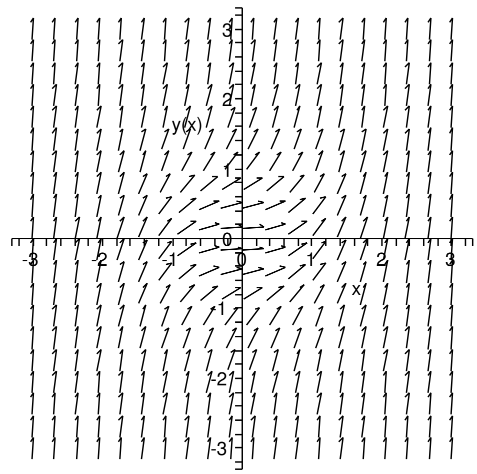
(a) 1.69
(b) 3.0
(c) 8.0
(d) 14.78
(e) 21.38
(f) 29.55
(g) 32
(h) 38.82
(i) 54

3. Find the slope field that for the differential equation

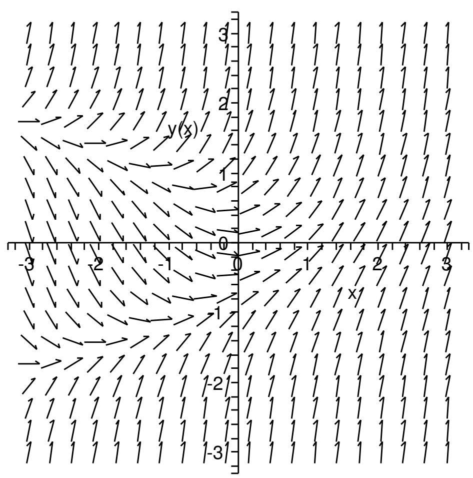
$$\frac{dy}{dx} = x + y^2$$



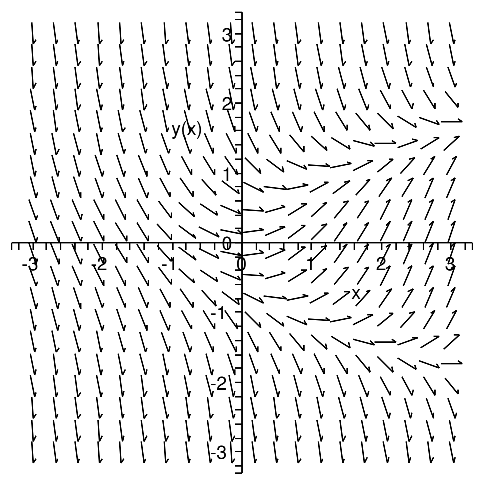
Plot (a)



Plot (b)



Plot (c)



Plot (d)

4.

$$y' = y^2 - x, \qquad y(0) = 1$$

Use Euler's method with step size 0.25 to approximate $y(0.5)$.

(a) 1.0
(b) 1.5
(c) 1.578125
(d) 1.667576
(e) 1.761821
(f) 1.774472
(g) 2.375
(h) 3.109275
(i) 5.3125

5. Find the correct form for a particular solution to the equation

$$y^{(4)} + 2y^{(3)} + y^{(2)} = x^2 + e^x$$

(a) $y_p = A + Bx + Cx^2 + De^x$
(b) $y_p = Ax + Bx^2 + Cx^3 + De^x$
(c) $y_p = Ax^2 + Bx^3 + Cx^4 + De^x$
(d) $y_p = A + Bx + Cx^2 + De^x + Exe^x$
(e) $y_p = Ax + Bx^2 + Cx^3 + De^x + Exe^x$
(f) $y_p = Ax^2 + Bx^3 + Cx^4 + De^x + Exe^x$

6. Transform the differential equation using the Laplace transform. Find the transform of the solution (i.e., solve for $X(s)$).

$$x'' - x' + x = 1, \qquad x(0) = 0, \quad x'(0) = -1$$

(a) $X(s) = \dfrac{1}{s(s^2 - s + 1)}$

(b) $X(s) = \dfrac{1}{s^2 - s + 1}$

(c) $X(s) = \dfrac{1 - s}{s(s^2 - s + 1)}$

(d) $X(s) = \dfrac{1 + s}{s(s^2 + s + 1)}$

(e) $X(s) = \dfrac{1}{s(s^2 + s + 1)}$

(f) $X(s) = \dfrac{s^2 - s - 1}{s(s^2 - s + 1)}$

(g) $X(s) = \dfrac{1}{s^2 + s + 1}$

(h) $X(s) = \dfrac{1}{s - s^2}$

(i) $X(s) = \dfrac{1}{s + s^2}$

(j) $X(s) = \dfrac{1}{s}$

7.

$$y'' + e^x y = 1 - x^2, \qquad y(0) = 1, y'(0) = 0$$

Find a power series solution in the form $y = \sum_{n=0}^{\infty} c_n x^n$.

Find $c_0 + 2c_1 - c_2 + 3c_3$.

(a) -3

(b) -2

(c) -1

(d) $-\dfrac{1}{2}$

(e) $-\dfrac{1}{6}$

(f) 0

(g) $\dfrac{1}{6}$

(h) $\dfrac{1}{2}$

(i) 2

(j) 3

8. Select the largest interval on which the initial value problem below has a unique solution.

$$y' = \frac{3y}{x}, \qquad y(2) = 4$$

(a) $(1, 3)$
(b) $(1, \infty)$
(c) $(0, 4)$
(d) $(-\infty, 0)$
(e) $(0, \infty)$
(f) $(-1, 5)$
(g) $(-1, \infty)$
(h) $(-\infty, \infty)$

9. Find the Laplace transform of the function $f(t) = |t|$.

$$\mathscr{L}\{|t|\} =$$

(a) 1
(b) $-\frac{1}{s}$
(c) $\frac{1}{s}$
(d) $-\frac{1}{s-1}$
(e) $\frac{1}{s-1}$
(f) $-\frac{1}{s^2}$
(g) $\frac{1}{s^2}$
(h) $-\frac{1}{(s-1)^2}$
(i) $\frac{1}{(s-1)^2}$
(j) $-\frac{1}{s^3}$
(k) $\frac{1}{s^3}$

10. Given that $y_1 = x$ and $y_2 = x^3$ are solutions of

$$y'' - \frac{3}{x}\, y' + \frac{3}{x^2}\, y = 0$$

Find the general solution of

$$y'' - \frac{3}{x}\, y' + \frac{3}{x^2}\, y = \frac{1}{x}$$

(a) $y = c_1 \ln x$

(b) $y = c_1 x + c_2 x^3$

(c) $y = c_1 x + c_2 x^3 - \frac{1}{2} \ln x$

(d) $y = c_1 x + c_2 x^3 - \frac{1}{2} x \ln x$

(e) $y = c_1 x + c_2 x^3 - \frac{1}{2} e^x$

(f) $y = c_1 x + c_2 x^3 + c_3 \ln x$

(g) $y = c_1 x + c_2 x^3 + c_3 x \ln x$

(h) $y = c_1 x + c_2 x^3 + c_3 e^x$

11. The following equations all have a singular point at $x = 0$. For which of the following is $x = 0$ a regular singular point?

    I. $xy'' + y' + (\sin x)y = 0$
    II. $x^2y'' + (e^x - 1)y' + (\sin x)y = 0$
    III. $x^3y'' + x^2y' + (\cos x)y = 0$
    IV. $x^4y'' + x^2(\sin x)y' + (\cos x - 1)y = 0$

    (a) I and II only
    (b) I and III only
    (c) I and IV only
    (d) II and III only
    (e) II and IV only
    (f) III and IV only
    (g) I, II and III only
    (h) I, II and IV only
    (i) II, III and IV only
    (j) All equations have $x = 0$ as a regular singular point

12. The point $x = 0$ is an ordinary point of

$$(x^2 - 4)y'' + 3xy' + y = 0.$$

If $y = \sum_{n=0}^{\infty} c_n x^n$ is a solution, find the recurrence relation for the coefficients c_n.

(a) $c_{n+2} = \dfrac{nc_n}{4(n+2)(n+1)}$

(b) $c_{n+2} = \dfrac{nc_{n+1} + c_n}{(n+2)(n+1)}$

(c) $c_{n+2} = \dfrac{(n+1)c_n}{n+2}$

(d) $c_{n+2} = \dfrac{(n+1)c_n + c_{n+1}}{4(n+1)}$

(e) $c_{n+2} = \dfrac{(n+1)c_n}{4(n+2)}$

(f) $c_{n+2} = \dfrac{nc_n}{4(n+2)}$

(g) $c_{n+2} = \dfrac{nc_n}{4(n+1)}$

(h) $c_{n+2} = \dfrac{nc_n}{(n+2)(n+1)}$

13. Let $y = \sum_{n=0}^{\infty} c_n x^n$ be the power series solution to the Airy equation

$$y'' - xy = 0, \qquad y(0) = 2, y'(0) = 0$$

What is c_6?

(a) $c_6 = \frac{1}{360}$

(b) $c_6 = \frac{1}{720}$

(c) $c_6 = \frac{1}{90}$

(d) $c_6 = \frac{1}{30}$

(e) $c_6 = \frac{1}{120}$

(f) $c_6 = \frac{1}{24}$

(g) $c_6 = \frac{1}{6}$

14. What value of α will exhibit the phenomenon of resonance

$$\alpha^2 y'' + y = \sin(2x).$$

(a) 0
(b) $\frac{1}{8}$
(c) $\frac{1}{4}$
(d) $\frac{1}{2}$
(e) $\frac{1}{\sqrt{2}}$
(f) $\sqrt{2}$
(g) 2
(h) 4
(i) 8
(j) No value of α will cause resonance

15. Consider the autonomous differential equation

$$x' = (x-1)^k(x-k).$$

Which value(s) of k will make $x = 1$ a stable critical point

I. $k = 1$	**II.** $k = 2$	**III.** $k = 3$	**IV.** $k = 4$

(a) I only
(b) II only
(c) III only
(d) IV only
(e) I and II
(f) I and III
(g) II and III
(h) I, II, and III
(i) II,III and IV
(j) I, III, and IV
(k) None of the values of k will make $x = 1$ a stable critical point.

16. Which of the following equations is exact

I. $(6xy - y^3)\,dx + (4y + 3x^2 - 3xy^2)dy = 0$

II. $(\cos x + \ln y)\,dx + \left(\dfrac{x}{y} + e^y\right)\,dy = 0$

III. $(2x + 3y)\,dx + (2xy + y^2)\,dy = 0$

IV. $y^2\,dx + 3xy\,dy = 0$

(a) I only
(b) II only
(c) III only
(d) IV only
(e) I and II only
(f) I and III only
(g) II and III only
(h) I, II, and III only
(i) II,III and IV only
(j) I, III, and IV only

17. Solve the system

$$\begin{aligned} x' &= x + 2y, \quad x(0) = 2 \\ y' &= 2x + y, \quad y(0) = 0 \end{aligned}$$

Find $x(1)$ and select the closest answer below.

(a) 14
(b) 16
(c) 18
(d) 20
(e) 22
(f) 24
(g) 26
(h) 28
(i) 30
(j) 32
(k) 34

18. Consider the differential equation

$$6x^2y'' + 7xy' - (x^2 + 2)y = 0$$

Which of the following are guaranteed to be solutions by the method of Frobenius.

I. $y = \sum_{n=0}^{\infty} c_n x^n$
II. $y = x^{-1/2} \sum_{n=0}^{\infty} c_n x^n$
III. $y = x^{-2/3} \sum_{n=0}^{\infty} c_n x^n$
IV. $y = x^{1/2} \sum_{n=0}^{\infty} c_n x^n$

(a) I only
(b) II only
(c) III only
(d) IV only
(e) I and II
(f) I and III
(g) II and III
(h) III and IV
(i) I, II, and III
(j) II,III and IV
(k) I, III, and IV