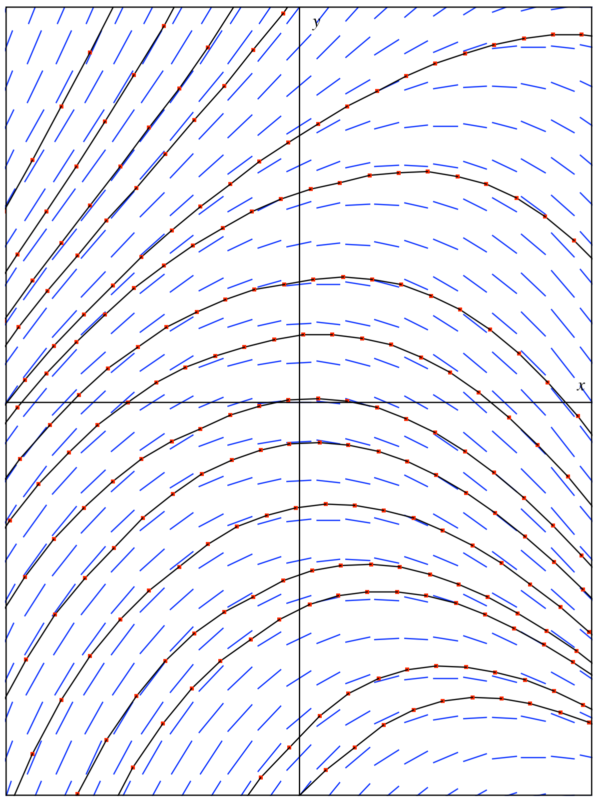
$\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$	$\mathscr{L}\left\{t^n\right\} = \dfrac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^a\right\} = \dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{\dfrac{1}{\sqrt{t}}\right\} = \dfrac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}\,dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathscr{L}\left\{\cosh(at)\right\} = \dfrac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \dfrac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \dfrac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \dfrac{a}{s^2+a^2}$
$\mathscr{L}\left\{\dfrac{1}{2a^3}(\sin at - at\cos at)\right\} = \dfrac{1}{(s^2+a^2)^2}$	$\mathscr{L}\left\{\dfrac{t}{2a}\sin at\right\} = \dfrac{s}{(s^2+a^2)^2}$
$\mathscr{L}\left\{\dfrac{1}{2a}(\sin at + at\cos at)\right\} = \dfrac{s^2}{(s^2+a^2)^2}$	$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{f''(t)\right\} = s^2\,F(s) - sf(0) - f'(0)$
$\mathscr{L}\left\{u_a(t)\right\} = \mathscr{L}\left\{u(t-a)\right\} = \dfrac{1}{s}e^{-sa}$	$\mathscr{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$
$\mathscr{L}\left\{\delta(t)\right\} = 1$	$\mathscr{L}\left\{\delta(t-a)\right\} = \mathscr{L}\left\{\delta_a(t)\right\} = e^{-sa}$
$\mathscr{L}\left\{\int_0^t f(u)\,du\right\} = \dfrac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\dfrac{1}{s}F(s)\right\} = \int_0^t f(u)\,du$
$(f*g)(t) = \int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{t\,f(t)\right\} = -F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^nF^{(n)}(s)$
$\mathscr{L}\left\{\dfrac{1}{t}f(t)\right\} = \int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\int_s^\infty F(u)\,du\right\}$
$\cosh t = \dfrac{1}{2}(e^t + e^{-t})$	$\sinh t = \dfrac{1}{2}(e^t - e^{-t})$

1. Find the differential equation that matches the given slope field



   (a) $y' = x^2 + y^2$
   (b) $y' = x - y^2$
   (c) $y' = x^2 - y^2$
   (d) $y' = x^2 - y$
   (e) $y' = y^2 - x$
   (f) $y' = y^2 + x$

2. The equation below has exactly two equilibrium solutions, $x = a$ and $x = b$. Find a and b and determine the stability of these solutions.

$$x' = (x - 5)(x^2 - 8x + 16)$$

   (a) $a + b = 9$ and both solutions are stable.
   (b) $a + b = 9$ and one solution is stable and the other is unstable.

3. Solve

$$xy' = 2y + x^3 \cos x$$

4. Suppose $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution to

$$y'' + (\sin x)y = e^x, \qquad y(0) = 0, y'(0) = -1$$

What is c_4?

5. The point $x = 0$ is a regular singular point to the equation below. Let a and b be the exponents of the of the differential equation at the singular point $x = 0$.

$$(x^2 - x)y'' + (\cos x + 1)y' + \frac{4}{x}y = 0$$

What is $a + b$?

6. Consider the differential equation

$$(x - 4)y'' + \frac{1}{x}y' + \frac{1}{\cos x}y = 0$$

Which of the following are true.

   I. The point $x = 0$ is a regular singular point of the equation (with exponents . . .)
   II. The point $x = 0$ is an irregular singular point of the equation
   III. The point $x = 0$ is an ordinary point of the equation

7. Solve

$$(x^2 - 1)y'' - 6xy + 12y = 0, \qquad y(0) = 1, y'(0) = 0$$

as a series solution of the form

$$y = \sum_{n=0}^{\infty} c_n x^n$$

8. Consider

$$4xy'' + 2y' + y = 0.$$

This equation has $x = 0$ as a regular singular point with exponents 0 and $\frac{1}{2}$ (you don't need to check this).

   I. There exists a solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$ (where the coefficients, a_n, can be determined).
   II. There exists a solution of the form $y = \sqrt{x}\sum_{n=0}^{\infty} b_n x^n$ (where the coefficients, b_n, can be determined).

9. Suppose $f(x)$ is a solution of $y' - 2y = x$, $y(0) = 1$. Find $L\{f\}(3)$

10. Which of the following statements about the differential equation $ydx + xdy = 0$ are true?

   (a) The ODE is exact

(b) The ODE is separable

(c) The ODE is first order linear

(d) The ODE is homogeneous

11. Consider the ODE $(y - x)y' = 1$. On which of the following rectangles does a unique solution exist?

    (a) $(0, 1) \times (0, 1)$

    (b) $(0, 2) \times (1, 2)$

    (c) $(1, 2) \times (1, 2)$

    (d) $(2, 3) \times (0, 1)$

    (e) $(1, 3) \times (0, 2)$

12. The autonomous equation $y' = (y - 4)^2(y + 2)(y - 3)(y - 5)$ has 4 equillibra points. Find the sum of the unstable ones.

    (a) -2

    (b) -1

    (c) 3

    (d) 5

    (e) 7

    (f) 8

    (g) 10

13. Let

$$f(t) = \begin{cases} 3 - t & t < 4 \\ 0 & t = 4 \\ t & t > 4 \end{cases}$$

    Compute $\mathscr{L}\{f\}(2)$.

14. Find $\mathscr{L}\{\sin^2(t)\}$

15. Which of the following 2nd order ODEs have ordinary points at $x = 0$?

    (a) $y'' + xy' + (\sin x)y = 0$

    (b) $y'' + xy' + (\cos x)y = 0$

    (c) $xy'' + xy' + (\sin x)y = 0$

    (d) $xy'' + xy' + (\cos x)y = 0$

16. Find a nontrivial lower bound for the radius of convergence of a power series solution of $(x^2 + 1)(x + 1)y'' + xy' + y = 0$ centered at $x = 1$.

17. What is the radius of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n.$$

(a) $\rho = 0$
(b) $\rho = 1/4$
(c) $\rho = 1/2$
(d) $\rho = 1$
(e) $\rho = 2$
(f) $\rho = 4$
(g) $\rho = \infty$

18. Consider the differential equation

$$x^2(x-1)y'' + (\sin^2 x)y' + (x^4 + x^2)y = 0.$$

I. $x = 0$ is an ordinary point.
II. $x = 0$ is an irregular singular point.
III. $x = 1$ is an ordinary point
IV. $x = 1$ is a regular singular point

(a) I only
(b) II only
(c) III only
(d) IV only
(e) I and II
(f) I and III
(g) II and III
(h) III and IV
(i) I, II, and III

19. (a) The Bessel function of order zero, denoted $J_0(t)$, is defined by the differential equation

$$tx'' + x' + tx = 0 \qquad x(0) = 1, x'(0) = 0.$$

Show that a solution is given by the everywhere convergent power series

$$J_0(t) = \sum_{n=0}^{\infty} c_n x^n.$$

(b) Either use the differential equation or the power series to show that

$$\mathscr{L}\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}},$$

(c) Let $f(t) = (J_0 * \cos)(t)$, calculate

$$\mathscr{L}\left\{\frac{f(t)}{t}\right\}.$$

20. Given that e^{2x} is a solution to the differential equation

$$y''' + y' - 10y = 0$$

find two other linearly independent solutions.

(a) $\{\cos x, \sin x\}$
(b) $\{e^x \cos x, e^x \sin x\}$
(c) $\{e^x \cos 2x, e^x \sin 2x\}$
(d) $\{e^{2x} \cos x, e^{2x} \sin x\}$
(e) $\{e^{-x} \cos x, e^{-x} \sin x\}$
(f) $\{e^{-x} \cos 2x, e^{-x} \sin 2x\}$
(g) $\{e^{-2x} \cos x, e^{-2x} \sin x\}$

21. Express the general solution of $\dfrac{dy}{dx} = 1 + 2xy$ in terms of the error function

$$E(x) = \int_0^x e^{-t^2}\, dt.$$

(a) $y(x) = CE(x)$
(b) $y = Ce^{x^2} E(x)$
(c) $y = e^{x^2}(C + E(x))$
(d) $y = Ce^{-x^2} E(x)$
(e) $y = e^{-x^2}(C + E(x))$
(f) $y = e^{-x^2}(C + 2E(x))$

22. Suppose $y = \sum_{n=0}^{\infty} c_n x^n$ is the solution to

$$(x-3)(x+2)y'' + y' - 2x^5 y = 0$$

and $\rho = \lim_{n\to\infty} \frac{|c_n|}{|c_{n+1}|}$. Determine which inequality for ρ is guaranteed by the theorem discussed in class and the text book.

(a) $\rho \geq 1$
(b) $\rho \geq 2$
(c) $\rho \geq 3$
(d) $\rho \geq 4$
(e) $\rho \geq 5$
(f) $\rho \geq 6$

23. Consider the differential equation modelling a mass-spring system.

$$31x'' + 16x' + 2x = 0.$$

   (a) This equation is an example of resonance
   (b) This equation is an example of overdamping
   (c) This equation is an example of underdamping
   (d) This equation is an example of critical damping
   (e) None of the above

24. Suppose a mass-spring system with no damping has a mass of $m = 3$, spring constant of $k = 12$, and external force of $F_E(t) = -6\sin(\alpha t)$. What value of α will exhibit the phenomenon of resonance.

25. Find all eigenfunctions of the problem

$$y'' + \lambda y = 0, \qquad y'(0) = 0,\ y(\pi) = 0.$$

26. Find the Frobenius series solutions of

$$2xy'' + 3y' - y = 0.$$