

Name:**ID:****Section:** ☐ **Moen** ☐ **Thornton**

This exam has:

- 14 multiple choice questions worth 5 points each.
- 2 hand graded questions worth 15 points each.

Important:

- No graphing calculators! Any non-graphing scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution as well as for your work.
- You are allowed both sides of 3×5 “cheat card” for the exam.
- For the written section, please make a check next to your instructor on both pages.

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$	$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$
$\mathcal{L}\{t^a\} = \frac{\Gamma(a+1)}{s^{a+1}}$	$\mathcal{L}\left\{\frac{1}{\sqrt{t}}\right\} = \frac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2 - a^2}$	$\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2 - a^2}$
$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2}$	$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$
$\mathcal{L}\left\{\frac{1}{2a^3}(\sin at - at \cos at)\right\} = \frac{1}{(s^2 + a^2)^2}$	$\mathcal{L}\left\{\frac{t}{2a} \sin at\right\} = \frac{s}{(s^2 + a^2)^2}$
$\mathcal{L}\left\{\frac{1}{2a}(\sin at + at \cos at)\right\} = \frac{s^2}{(s^2 + a^2)^2}$	$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$	$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
$\mathcal{L}\{u_a(t)\} = \mathcal{L}\{u(t-a)\} = \frac{1}{s}e^{-sa}$	$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$
$\mathcal{L}\{\delta(t)\} = 1$	$\mathcal{L}\{\delta(t-a)\} = \mathcal{L}\{\delta_a(t)\} = e^{-sa}$
$\mathcal{L}\left\{\int_0^t f(u) du\right\} = \frac{1}{s}F(s)$	$\mathcal{L}^{-1}\left\{\frac{1}{s}F(s)\right\} = \int_0^t f(u) du$
$(f * g)(t) = \int_0^t f(u)g(t-u) du$	$\mathcal{L}\{(f * g)(t)\} = F(s)G(s)$
$\mathcal{L}\{tf(t)\} = -F'(s)$	$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$
$\mathcal{L}\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(u) du$	$\mathcal{L}^{-1}\{F(s)\} = t\mathcal{L}^{-1}\left\{\int_s^\infty F(u) du\right\}$
$\cosh t = \frac{1}{2}(e^t + e^{-t})$	$\sinh t = \frac{1}{2}(e^t - e^{-t})$

1. Given the differential equation:

$$x^{(3)} + x'' + (\sin t)x' + x^2 + t^2 = e^{t^2}$$

select the system of differential equations that is equivalent to the equation above.

- (a) $x'_1 = x_2$
 $x'_2 = x_3 + (\sin t)x_2$
 $x'_3 = e^{t^2} - (x_1^2 + t^2)$
- (b) $x'_1 = x_1$
 $x'_2 = x_2$
 $x'_3 = e^{t^2} + (\sin t)x_2 - (x_1^2 + t^2)$
- (c) $x'_1 = x_1^2 + t^2$
 $x'_2 = (\sin t)x_2$
 $x'_3 = e^{t^2}$
- (d) $x'_1 = x_1^2 + t^2$
 $x'_2 = x_3 + (\sin t)x_1$
 $x'_3 = x_3$
- (e) $x'_1 = x_1$
 $x'_2 = x_2$
 $x'_3 = e^{t^2} - (x_1^2 + t^2) - (\sin t)x_2$
- (f) $x'_1 = x_2$
 $x'_2 = x_3$
 $x'_3 = e^{t^2} - (x_1^2 + t^2) - (\sin t)x_2 - x_3$

2. Find the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{1}{\sqrt{s+1}}\right\}$$

(a) $\sqrt{\pi t}$

(b) $e^{-t}\sqrt{\pi t}$

(c) $\frac{e^{-t}}{\sqrt{\pi t}}$

(d) $e^t\sqrt{\pi t}$

(e) $e^{-t}\sqrt{\frac{\pi}{t}}$

(f) $e^t\sqrt{\frac{t}{\pi}}$

(g) $\frac{1}{\sqrt{\pi t}}$

(h) $\sqrt{\frac{\pi}{t}}$

(i) $\sqrt{\frac{t}{\pi}}$

3. Suppose y satisfies the initial value problem

$$ty'' + y' = -2, \quad y(0) = 0, y'(0) = 0,$$

and $Y(s) = \mathcal{L}\{y(t)\}$. What differential equation does Y satisfy?

(a) $Y' + \frac{1}{s}Y = \frac{2}{s}$

(b) $Y' + \frac{1}{s}Y = \frac{2}{s^2}$

(c) $Y' + \frac{1}{s}Y = \frac{2}{s^3}$

(d) $Y' - \frac{1}{s}Y = \frac{2}{s^3}$

(e) $Y' + Y = \frac{2}{s^3}$

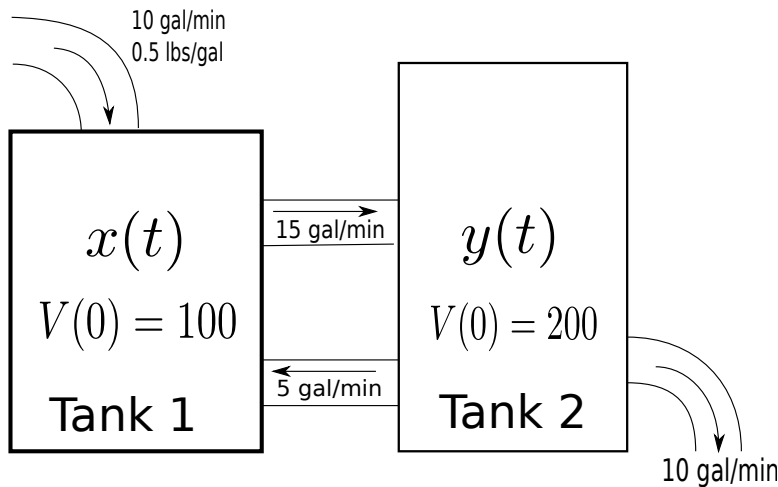
(f) $Y' + sY = 2$

(g) $Y' + sY = \frac{2}{s}$

(h) $Y'' - \frac{1}{s}Y = \frac{2}{s^2}$

4. Two tanks filled with brine are connected together as in the figure below. Tank 1, with $x(t)$ lbs of salt in it, starts with 100 gallons of brine. Tank 2, with $y(t)$ lbs of salt in it, starts with 200 gallons of brine. Brine at concentration 0.5 lbs/gal flows into Tank 1 at a rate of 10 gal/min. Brine is pumped from Tank 1 to Tank 2 at a rate of 15 gal/min. Brine is pumped from Tank 2 to Tank 1 at a rate of 5 gal/min. Brine flows from Tank 2 at a rate of 10 gal/min. As usual, we assume that the tanks are completely mixed.

Which of the following systems of equations models this system?



- (a) $x' = -15x + 5y + 5$
 $y' = 15x - 5y$
- (b) $x' = -15x + 5y$
 $y' = 15x - 5y$
- (c) $40x' = -6x + y + 200$
 $40y' = 6x - 3y$
- (d) $40x' = 5x - y + 5$
 $40y' = 15x - 5y$
- (e) $40x' = -15x + 5y + 5$
 $40y' = 15x - 5y$
- (f) $40x' = -15x + 5y + 5$
 $40y' = 15x - 5y$
- (g) $40x' = -6x + y + 5$
 $40y' = 6x - 3y$

5. Find the inverse Laplace transform of

$$\frac{s+8}{s^3+4s}$$

- (a) $1 - \sin 2t + 2 \cos 2t$
- (b) $-1 + \frac{1}{2} \sin 2t + 2 \cos 2t$
- (c) $2 + \sin 2t + 2 \cos 2t$
- (d) $2 + \frac{1}{2} \sin 2t + 2 \cos 2t$
- (e) $2 + \frac{1}{2} \sin 2t - 2 \cos 2t$
- (f) $2 - \frac{1}{2} \sin 2t + 2 \cos 2t$
- (g) $2 - \frac{1}{2} \sin 2t - 2 \cos 2t$
- (h) $2 + \sin 2t + \frac{1}{2} \cos 2t$
- (i) $2 + \sin 2t - \frac{1}{2} \cos 2t$
- (j) $2 - \sin 2t + \frac{1}{2} \cos 2t$
- (k) $2 - \sin 2t - \frac{1}{2} \cos 2t$

6. Suppose that y is a smooth function and $Y(s) = \mathcal{L}\{y(t)\}$ satisfies

$$\mathcal{L}\{y\}(3) = 1, \quad y(0) = 2, \quad \text{and} \quad y'(0) = 1.$$

What is the value of $\mathcal{L}\{y''\}(3)$?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4
- (f) 5

7. Suppose $\mathbf{A}$ is a 2×2 matrix that satisfies

$$\mathbf{A} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{A} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Find the general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

- (a) $\mathbf{x}(t) = c_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$
- (b) $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}$
- (c) $\mathbf{x}(t) = c_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$
- (d) $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t$
- (e) $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$
- (f) $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}$

8. Suppose that $F(s) = \mathcal{L}\{f(t)\}$ and that

$$\mathcal{L}^{-1}\{F'(s)\} = \arctan(t).$$

What is $f(t)$?

- (a) $f(t) = \frac{-1}{t(t^2 + 1)}$
- (b) $f(t) = -t \arctan t$
- (c) $f(t) = \int_0^t \arctan(\tau) d\tau$
- (d) $f(t) = -\frac{1}{t} \arctan(t)$
- (e) $f(t) = \frac{1}{t^2 + 1}$
- (f) $f(t) = \frac{-t}{t^2 + 1}$

9. Let $f(t)$ be given by

$$f(t) = \begin{cases} 3 & \text{if } 2 < t < 6 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\mathcal{L}\{f(t)\}$.

- (a) $\frac{1}{s}(e^{-2s} - e^{-6s})$
- (b) $\frac{1}{s}(e^{-2} - e^{-6})$
- (c) $\frac{1}{s}(e^{-6s} - e^{-2s})$
- (d) $\frac{1}{s}(e^{-6} - e^{-2})$
- (e) $\frac{3}{s}(e^{-2s} - e^{-6s})$
- (f) $\frac{3}{s}(e^{-2} - e^{-6})$
- (g) $\frac{3}{s}(e^{-6s} - e^{-2s})$
- (h) $\frac{3}{s}(e^{-6} - e^{-2})$

10. Let λ_1 and λ_2 be the eigenvalues of the matrix A .

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

What is $|\lambda_1 - \lambda_2|$?

- (a) 2
- (b) 3
- (c) 4
- (d) 5
- (e) 6
- (f) 7
- (g) 8
- (h) 9
- (i) 10

11. Let A be the product of the matrices:

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} -3 & -1 & 2 \\ -1 & 0 & 7 \\ 2 & 1 & 0 \end{bmatrix}$$

Find the entry in the matrix A that is in the middle middle of the top row (row 1, column 2)?

- (a) -4
- (b) -3
- (c) -2
- (d) -1
- (e) 0
- (f) 1
- (g) 2
- (h) 3
- (i) 4

12. Let $F(s)$ be the Laplace transform of $f(t) = 3 \sin 2t - \cosh 3t$. What is $F(4)$?
(Select the closest answer.)
- (a) -0.4202
 - (b) -0.2714
 - (c) -0.2435
 - (d) -0.1056
 - (e) -0.0453
 - (f) 0
 - (g) 0.0194
 - (h) 0.3297
13. Let $f(t)$ be the inverse Laplace transform of $F(s) = \frac{2s}{(s-1)^2 + 4}$. What is $f(2)$?
- (a) -18.23
 - (b) -15.25
 - (c) -11.93
 - (d) -7.13
 - (e) -0.745
 - (f) 0
 - (g) 3.68
 - (h) 5.43
 - (i) 7.12

14. Solve the differential equation

$$x'' + 4x = e^t \sqrt{t}, \quad x(0) = x'(0) = 0$$

- (a) $x = \sin t$
- (b) $x = \sin 2t$
- (c) $x = (\sin t) * (e^t \sqrt{t})$
- (d) $x = (\sin 2t) * (e^t \sqrt{t})$
- (e) $x = \left(\frac{1}{2} \sin 2t\right) * (e^t \sqrt{t})$
- (f) $x = (2 \sin 2t) * (e^t \sqrt{t})$
- (g) $x = (\cos t) * (e^t \sqrt{t})$
- (h) $x = (\cos 2t) * (e^t \sqrt{t})$
- (i) $x = \left(\frac{1}{2} \cos 2t\right) * (e^t \sqrt{t})$
- (j) $x = (2 \cos 2t) * (e^t \sqrt{t})$

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Note: You will be graded on the readability of your work. Use the back of this sheet, if necessary.

15. Solve the following initial value problem.

$$x'' + 2x' - 15x = 8\delta(t - 6) \quad x(0) = 1, x'(0) = 0$$

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Note: You will be graded on the readability of your work. Use the back of this sheet, if necessary.

16. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$