(You will be given this table on the exam.)

$\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$	$\mathscr{L}\left\{t^n\right\} = \dfrac{n!}{s^{n+1}}$
$\mathscr{L}\left\{t^a\right\} = \dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathscr{L}\left\{\dfrac{1}{\sqrt{t}}\right\} = \dfrac{\sqrt{\pi}}{\sqrt{s}}$
$\Gamma(x) = \displaystyle\int_0^\infty e^{-t}t^{x-1}\,dt$	$\Gamma(1/2) = \sqrt{\pi}$
$\mathscr{L}\left\{\cosh(at)\right\} = \dfrac{s}{s^2-a^2}$	$\mathscr{L}\left\{\sinh(at)\right\} = \dfrac{a}{s^2-a^2}$
$\mathscr{L}\left\{\cos(at)\right\} = \dfrac{s}{s^2+a^2}$	$\mathscr{L}\left\{\sin(at)\right\} = \dfrac{a}{s^2+a^2}$
$\mathscr{L}\left\{\dfrac{1}{2a^3}(\sin at - at\cos at)\right\} = \dfrac{1}{(s^2+a^2)^2}$	$\mathscr{L}\left\{\dfrac{t}{2a}\sin at\right\} = \dfrac{s}{(s^2+a^2)^2}$
$\mathscr{L}\left\{\dfrac{1}{2a}(\sin at + at\cos at)\right\} = \dfrac{s^2}{(s^2+a^2)^2}$	$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a)$
$\mathscr{L}\left\{f'(t)\right\} = s\,F(s) - f(0)$	$\mathscr{L}\left\{f''(t)\right\} = s^2\,F(s) - sf(0) - f'(0)$
$\mathscr{L}\left\{u_a(t)\right\} = \mathscr{L}\left\{u(t-a)\right\} = \dfrac{1}{s}e^{-sa}$	$\mathscr{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$
$\mathscr{L}\left\{\delta(t)\right\} = 1$	$\mathscr{L}\left\{\delta(t-a)\right\} = \mathscr{L}\left\{\delta_a(t)\right\} = e^{-sa}$
$\mathscr{L}\left\{\displaystyle\int_0^t f(u)\,du\right\} = \dfrac{1}{s}F(s)$	$\mathscr{L}^{-1}\left\{\dfrac{1}{s}F(s)\right\} = \displaystyle\int_0^t f(u)\,du$
$(f*g)(t) = \displaystyle\int_0^t f(u)g(t-u)\,du$	$\mathscr{L}\left\{(f*g)(t)\right\} = F(s)G(s)$
$\mathscr{L}\left\{t\,f(t)\right\} = -F'(s)$	$\mathscr{L}\left\{t^n\,f(t)\right\} = (-1)^nF^{(n)}(s)$
$\mathscr{L}\left\{\dfrac{1}{t}f(t)\right\} = \displaystyle\int_s^\infty F(u)\,du$	$\mathscr{L}^{-1}\left\{F(s)\right\} = t\mathscr{L}^{-1}\left\{\displaystyle\int_s^\infty F(u)\,du\right\}$
$\cosh t = \dfrac{1}{2}(e^t + e^{-t})$	$\sinh t = \dfrac{1}{2}(e^t - e^{-t})$

1. Write the system
$$\begin{cases} x'' = y - 2x \\ y' = x + 2y \end{cases}$$
as a linear differential equation.

2. Solve the initial value problem
$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

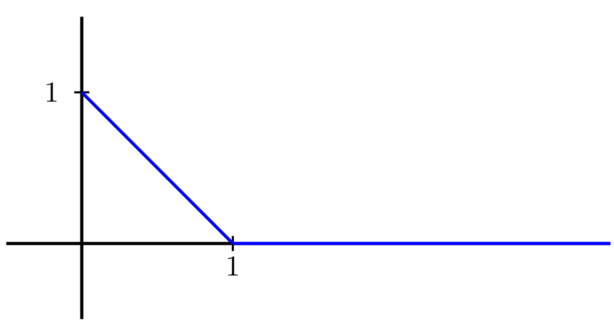
3. Suppose $F(s) = \mathscr{L}\{\sqrt{t^2+1}\}$, calculate
$$\mathscr{L}^{-1}\Big\{\frac{F'(s)}{s}\Big\}.$$

4. Let $\mathbf{x}$ be the solution to
$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
What is $\mathbf{x}(\pi)$?

5. The Bessel function of order 0, denoted $J_0(t)$, can be define for $t \geq 0$ via the Laplace transform as
$$\mathscr{L}\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}.$$
What is $(J_0 * J_0)(t)$?

6. Solve the system
$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 5 \\ 2 & 1 & -2 \end{bmatrix} \mathbf{x}$$

7. Let $f(t)$ be given by the graph



Calculate $\mathscr{L}\{f(t)\}$, as well as its domain.

8. The matrix
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$
has eigenvalues -1 and 3. Find the solution to the equation $\mathbf{x}' = \mathbf{A}\mathbf{x}$?

9. Use the Laplace transform to solve

$$y'' - 3y' + 2y = \sin t, \qquad y(0) = 1, y'(0) = 2$$

10. Solve the differential equation

$$tx'' - 2x' + tx = -\cos t, \qquad x(0) = x'(0) = 0.$$

(Hint: If $X(s) = \mathscr{L}\{x(t)\}$ then X satisfies a linear first order differential equation).

11. Solve the system

$$\begin{aligned} x_1' &= 2x_1 - x_2 \\ x_2' &= 3x_1 + x_2 \end{aligned}$$

12. If x and y are solutions of $x' = x + 3y$, $y' = x + 4y$, $x(0) = 2$, $y(0) = 1$, find $x(1)$.

13. Find the product of the eigenvalues of $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & -2 \end{bmatrix}$

14. Compute $\mathscr{L}\left\{\dfrac{e^t - \cos t}{t}\right\}$.

15. Let $f(t) = \mathscr{L}^{-1}\left\{\dfrac{-3s^2 + 3s + 4}{s^3 + 4s^2}\right\}$. Compute $f(0)$.

16. A mass $m = 1$ is attached to a spring with constant $k = 4$. Assume no damping force is present and the mass lie at rest in its equilibrium position. At time $t = 0$, the mass is hit with a hammer with a unit impulse in the positive direction.

    (a) Give an expression for the motion of the mass as a function of time

    (b) List all times between 0 and 3π that the mass is at its equillbrium position

    At time $t = 3\pi$, the mass is once again struck with a unit impulse in the positive direction.

    (c) List all times after 3π that the mass is at its equillibrium position.

    (d) Sketch the motion of the mass from $t = 0$ onwards