Name:
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This exam has 16 questions:

- 14 multiple choice questions worth 5 points each.
- 2 hand graded questions worth 15 points each.

Important:

- No graphing calculators! Any non-graphing scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution as well as for your work.
- You are allowed both sides of 3×5 note "cheat card" for the exam.

1. Consider the initial value problem

$$\frac{dy}{dx} = 2xy + 1 \qquad y(0) = 1$$

Approximate $y(1)$ using Euler's method with step size $h = 0.5$

(a) 0.25
(b) 1.0
(c) 1.5
(d) 1.75
(e) 2.0
(f) 2.25
(g) 2.5
(h) 2.75
(i) 3.0
(j) 3.25
(k) None of the above

2. A small population of rabbits, $R(t)$, is modeled by the differential equation

$$\frac{dR}{dt} = \frac{1}{10^5} R^2(R-35)(R-57), \qquad R(0) = 50$$

Determine $\lim_{t\to\infty} R(t)$.

(a) 0
(b) 35
(c) 50
(d) 57
(e) 1995
(f) 10^5
(g) ∞
(h) None of the above

3. If y is a solution to

$$\frac{dy}{dx} = x(2-y)^2, \quad y(1) = 2$$

what is $y(2)$?

(a) -3
(b) -2
(c) -1
(d) 0
(e) 1
(f) 2
(g) 3

4. Consider the differential equation below

$$y' = \frac{(y-1)^{2/3}}{x}$$

Determine which of the initial values will guarantee the existence of a unique solution for x near that initial value.

(a) $y(0) = 0$
(b) $y(0) = 1$
(c) $y(1) = 0$
(d) $y(1) = 1$
(e) $y(2) = 1$
(f) $y(0) = 2$
(g) None of the above guarantee a unique solution

5. Consider the initial value problem

$$xy' - 2y = 0, \quad y(1) = 1$$

Which of the following statements are true?

I. The equation has no solution
II. The equation has a unique solution defined on $(0, 2)$
III. The equation has a unique solution defined on $\mathbb{R}$

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I and III only
(f) II and III only
(g) All statements are true
(h) None of the statements are true

6. Which of the following differential equations is separable?

   (a) $y' - e^y = e^{x+y}$

   (b) $x\dfrac{dy}{dx} - 2y = x + 2y$

   (c) $xy\dfrac{dy}{dx} = x^3 + y^3$

   (d) $(x + y)y' = x - y$

   (e) $y' = \ln(xy)$

   (f) None of the above are separable

7. Which of the equations below is exact?

   I. $-\dfrac{y}{x^2}\,dx + \dfrac{1}{x}\,dy = 0$

   II. $(2xy - e^x) + x^2\,\frac{dy}{dx} = 1$

   III. $x\,dx - y\,dy = 0$

   IV. $y\,dx - x\,dy = 0$

   (a) I only

   (b) II only

   (c) III only

   (d) IV only

   (e) I and II only

   (f) I and III only

   (g) I and IV only

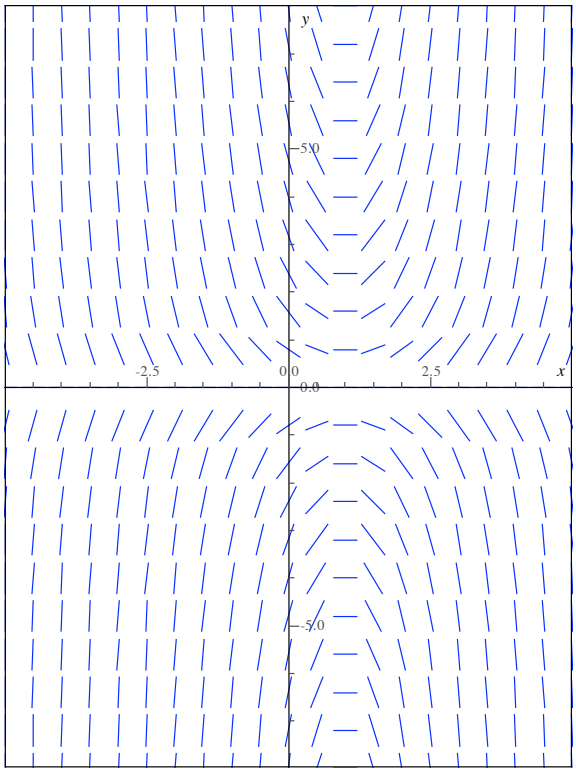
   (h) II and III only

   (i) II and IV only

   (j) III and IV only

   (k) Some other combination or none are exact

8. Pick the differential equation that corresponds to the given slope field.



(a) $y' = yx$

(b) $y' = y(x+1)$

(c) $y' = y(x-1)$

(d) $y' = (y+1)x$

(e) $y' = (y+1)(x+1)$

(f) $y' = (y+1)(x-1)$

(g) $y' = (y-1)x$

(h) $y' = (y-1)(x+1)$

9. Torricelli's Law can be formulated as a statement about the rate that water drops in a water tank as the water drains from a hole in the tank. This formulation of Torricelli's law is

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

where y is the height of the water, $A(y)$ is the cross-sectional area of the tank at height y, $g = 32$ ft/s^2 (acceleration due to gravity), and a is the area of the hole through which the water is draining.

An upright cylindrical water tank with a radius of 1 ft and a height of 4 ft and at time $t = 0$ is full of water. At that moment a circular hole with radius 1 inch (1/12 ft) is punched in the bottom of the tank. How long will it take for all of the water to drain from the tank?

(a) $t = 8$ s

(b) $t = 12$ s

(c) $t = 24$ s

(d) $t = 32$ s

(e) $t = 64$ s

(f) $t = 72$ s

10. Solve the initial value problem

$$\frac{dy}{dx} - \frac{2}{x}y = x^2e^x, \qquad y(1) = e$$

What is $y(2)$? Round your answer to one decimal place.

(a) 2.7
(b) 5.1
(c) 20
(d) 21.7
(e) 25.1
(f) 29.6

11. Solve the initial value problem

$$\frac{dy}{dx} = x\sqrt{x^2+9}, \qquad y(-4) = 0.$$

Find $y(0)$. Round your answer to one decimal place.

(a) -33.7
(b) -32.7
(c) -31.7
(d) 31.7
(e) 32.7
(f) 33.7

12. A 200L tank contains 10 kg of salt dissolved in 100L of water. Pure water is pumped into the tank at the rate of 3 L/s, and the uniform mixture is pumped out at a rate of 2 L/s. How much salt is in the tank when it is full? Pick the closest answer.

    (a) 0 kg

    (b) 1 kg

    (c) 1.5 kg

    (d) 2 kg

    (e) 2.5 kg

    (f) 3 kg

    (g) 3.5 kg

    (h) 4 kg

13. Consider the equation

$$2x - 2yy' = 0$$

Exactly one solution curve (trajectory) passes through the point $(2, 1)$. Which of the following points is also on that solution curve (trajectory)?

(a) $(-1, -2)$
(b) $(-1, -1)$
(c) $(0, 0)$
(d) $(1, 1)$
(e) $(1, 2)$
(f) $(\sqrt{2}, \sqrt{5})$
(g) $(\sqrt{5}, \sqrt{2})$
(h) None of these line on the same solution curve as the point $(2, 1)$

14. Find the solution to

$$y' = \frac{xy - y^2}{x^2}, \quad y(1) = \frac{1}{2}$$

Find $y(2)$ and round your answer to two decimal places.

(a) 0
(b) 0.21
(c) 0.53
(d) 0.67
(e) 0.70
(f) 0.74
(g) 0.83
(h) 0.94
(i) 1.0

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Note: You will be graded on the readability of your work. Use the back of this sheet, if necessary.

15. Consider the autonomous equation $x'(t) = \dfrac{2x^4 - 3x^3 - 20x^2}{50}$

    (a) Find all the equilibrium solutions.

    (b) Sketch the slope field corresponding to this equation. Include several possible solution curves.

    (c) For each solution that you found in part (a), state whether it is stable, unstable, or semistable.

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16. A tank initially contains 60 gal of pure water. Brine containing 1 lb/gal of salt enters the tank at 1 gal/min, and the well mixed solution leaves the tank at 2 gal/min.

    (a) How long does it take for the tank to empty?

    (b) Find the amount of salt, $x(t)$ in the tank after t minutes.

    (c) What is the maximum amount of salt ever in the tank?