Math 217
Practice Exam 1

1. Which of the following differential equations is exact?

   (a) $(3xy - x^3)dx + (3xy - 3x^2y)\ dy = 0$

   (b) $\sin(xy)\ dx + \cos(xy)\ dy = 0$

   (c) $xy^2 - x^2yy' = 0$

   (d) $(1 + ye^{xy}) + (2y + xe^{xy})\dfrac{dy}{dx} = 0$

   (e) $e^{-xy}\ dx - e^{xy}\ dy = 0$

   (f) $(2x^2y + y^2) + (2xy^2 + x^2)y' = 0$

   (g) $\frac{x}{y}dx + \frac{x^2}{2}\ln(y)dy = 0$

2. Suppose a population of deer $P(t)$ satisfies the initial value problem

$$\frac{dP}{dt} = 2(P - 1000)(5000 - P)^2, \quad P(0) = 2000.$$

   What is $\lim_{t\to\infty} P(t)$?

   (a) 0

   (b) 1000

   (c) 2000

   (d) 5000

   (e) 20000

   (f) ∞

3. Suppose a water tank initially contains a 100 gal mixture of salt and water. Brine at a concentrate of 0.5 lbs/gal flows into the tank at the same rate the well mixed brine flows out of the tank. How much salt should initially be in the tank so that the amount of salt stays constant as the brine flows through the tank, i.e., so equilibrium is acheived?

   (a) 100 lbs

   (b) 200 lbs

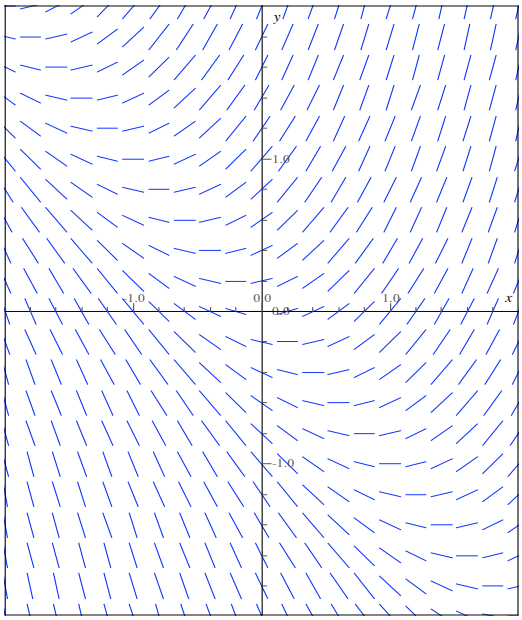
   (c) 150 lbs

   (d) 250 lbs

   (e) 50 lbs

   (f) 0 lbs

4. Which of the following differential equations best represents the slope field below?



(a) $y' = x + y$

(b) $y' = x - y$

(c) $y' = y - x$

(d) $y' = y - x + 1$

(e) $y' = y - x + 2$

(f) $y' = x - y + 1$

5. Consider the following differential equation

$$\frac{dx}{dt} = kx^2(3-x)(x-1).$$

Which of the following are true?

I. If $k > 0$ then $x = 3$ is a stable critical point.

II. If $k > 0$ then $x = 1$ is a stable critical point.

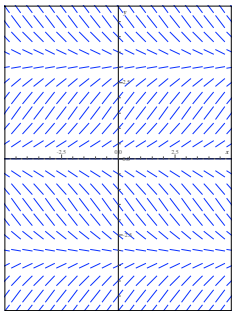
III. If $k < 0$ then $x = 1$ is a stable critical point.

(a) I only
(b) II only
(c) III only
(d) I and II
(e) I and III
(f) II and III
(g) All are true
(h) None are true

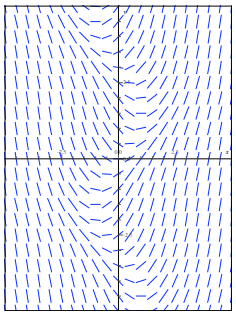
6. Identify the slope field of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^2 + \sin y.$$

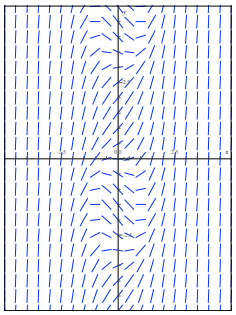
(a)



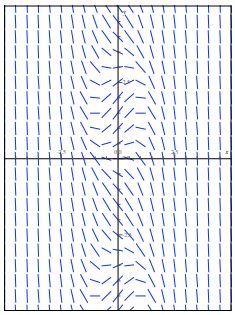
(b)



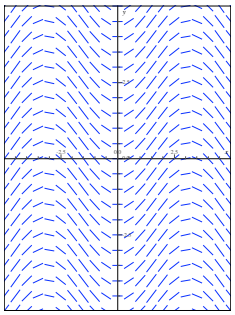
(c)



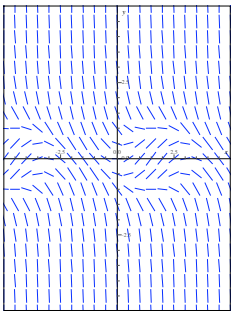
(d)



(e)



(f)



7. Consider the table of value for the two variable function $f(x, y)$.

(x, y)	$f(x, y)$
$(0, 1)$	-1
$(1, 0)$	1
$(1, 1)$	2
$(2, 1)$	5

If y_n is the n-th approximate using the improved Euler method with step size $h = 1$ of the IVP

$$\frac{dy}{dx} = f(x, y), \quad y(0) = 1$$

what is y_3?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5
(g) 6
(h) 7
(i) None of the above

8. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x - 1}{y^2} \qquad y(0) = 3$$

What is $y(2)$?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5
(g) 6
(h) None of the above

9. Consider the differential equations below. Determine which are guaranteed a unique solution near the initial value according to theorems discussed in class and the textbook.

   I. $y' = (x-1)(y-2)^{2/3} \qquad y(1) = 1$
   II. $y' = (x-1)(y-2)^{2/3} \qquad y(1) = 2$
   III. $y' = (x-1)(y-2)^{2/3} \qquad y(2) = 2$
   IV. $y' = (x-1)(y-2)^{2/3} \qquad y(2) = 1$

   (a) Only I has a unique solution
   (b) Only II has a unique solution
   (c) Only III has a unique solution
   (d) Only IV has a unique solution
   (e) Only I and II have unique solutions
   (f) Only I and III have unique solutions
   (g) Only I and IV have unique solutions
   (h) Only II and III have unique solutions
   (i) Only II and IV have unique solutions
   (j) Only III and IV have unique solutions
   (k) None of the above.

10. Solve the initial value problem

$$\frac{dy}{dx} = \frac{y}{x}(\ln y - \ln x + 1) \qquad y(1) = 2$$

What is $y(2)$?

   (a) 0
   (b) 2
   (c) 4
   (d) 6
   (e) 8
   (f) 10
   (g) None of the above

11. Find the general solution to

$$(2x^3y + 4xy^3)\frac{dy}{dx} = 3x^2y^2 + 5y^4,$$

(a) $y^4 + y^2 = Cx$
(b) $y^4 + x^2y^2 = Cx^3$
(c) $x^4y^4 + x^2y^2 = Cx^4e^x$
(d) $y^4 + x^2y^4 = Cx^2e^x$
(e) $y^4 + x^2y^2 = Cx^5$
(f) $y^2 + y^2 = Cx^5$

12. Which of the following equations has $y = 1$ as a stable critical point?

I) $y' = y^2 - 1$,
II) $y' = y - 1$,
III) $y' = y^2 - 5y + 4$.

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I and III only
(f) II and III only
(g) I, II, and III

13. At noon a car starts from rest at point A and proceeds at constant acceleration along a straight road toward point B. If the car reaches B at 12:50 P.M. with a velocity of 60 mi/h, what is the distance from A to B?

(a) 25 miles
(b) 27 miles
(c) 29 miles
(d) 31 miles
(e) 33 miles
(f) 35 miles

14. Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = \frac{\cos x}{e^x} \qquad y(0) = 1.$$

What is $y(\pi)$?

(a) $2e^{\pi}$
(b) $e^{\pi} + 1$
(c) e^{π}
(d) $\frac{2}{e^{\pi}}$
(e) $\frac{1}{e^{\pi}} + 1$
(f) $\frac{1}{e^{\pi}}$

15. Solve the differential equation

$$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 3y^2)\frac{dy}{dx} = 0$$

(a) $y\cos x + 2xe^y = C$
(b) $\sin x + x^2e^y - 3y^2 = C$
(c) $y^2\cos x + 2xe^y = C$
(d) $y\sin x + x^2e^y - 3xy^2 = C$
(e) $y\sin x + 2xe^y - y^3 = C$
(f) $y\cos x + x^2e^y - y^3 = C$
(g) $y\sin x + x^2e^y - 3y^2 = C$
(h) $y\cos x + 2xe^y - y^3 = C$

Written Problem: Show your work.

16. Consider the following autonomous equation:

$$y' = y^6 - 2y^5 + 2y^3 - y^2$$

(a) Find all equillibria

(b) Sketch some solution curves (enough to infer the limiting behavior from any initial condition)

(c) Classify all equillibria

Written Problem: Show your work.

17. A 100L tank has 50L of a 2 kg/L salt solution in it. Pure water is flowing in to the tank at 10L/min, and the well mixed solution is draining from the tank at 5L/min.

    (a) At what time does the tank overflow?

    (b) Find the concentration of the solution in the tank at the time it overflows.

    (c) After the tank starts to overflow, we continue to pump in pure water at 10 L/min. Since the tank is overflowing, solution is now leaving the tank at the same rate as water is being pumped in. What is the concentration of the solution if we do this for 10 minutes after it overflows?