Math 217 - December 10, 2010
Solutions

1. Solve

   (a) $y'' - 4y' + 5y = 16x\cos x, \quad y(0) = y'(0) = 0$
   Solution:
   $$y = e^{2\,x}\,\left(2\,sin\,(x) - cos\,(x)\right) + (-2\,x - 2)\,\,sin\,(x) + (2\,x + 1)\,\,cos\,(x)$$

   (b) $2y'' + 5y' - 3y = 27x^2, \quad y(0) = 1, y'(0) = -1$ (this one doesn't work out too nicely)
   Solution:
   $$y = \frac{436\,e^{\frac{x}{2}}}{7} + \frac{5\,e^{-3\,x}}{7} - 9\,x^2 - 30\,x - 62$$

   (c) $y^{(6)} - 2y^{(5)} + y^{(4)} = x^2$

   (d) $x''' - x'' + x' - x = 0$
   Solution: $x = c_1 e^t + c_2\cos t + c_3 \sin t$

   (e) $xy' = 217 - (x+1)y, y(1) = 217.$
   Solution: $y = 217/x$

   (f) $x'' - 6x' + 9x = 4e^{2t}, x(0) = -1, x'(0) = 1.$
   Solution: $x = (8t - 5)e^{3t} + 4e^{2t}$

   (g) $xy^2y' = x^3 + y^3, y(1) = 0.$
   Solution: Homogeneous, $y = x\sqrt[3]{3\ln x}$

2. Find the singular points and the exponents of $xy'' + 2y' - y = 0.$

   Solution: $x = 0$ is singular, indicial equation: $r(r-1) + 2r = 0$

3. Find the recurrance relation for the frobenius solution with exponent 1/2 for $2x^2y'' + 3xy' - (x^2+1)y = 0$

4. The general solution of $y'' - 2y/x^2 = 0$ is $y = Ax^2 + B/x$. Find a solution to $y'' - 2y/x^2 = 6.$

   Solution: $y = 2x^2\ln x - 2x^2/3$

5. Identify all ordinary and singular points to $x(x-1)y'' + (\sin x)y' + xy = 0$

6. Solve with power series centered at $x = 0$: $(2 - x^2)y'' - xy' + 16y = 0, y(0) = 1, y'(0) = 0$