

Math 217 - December 9, 2010

1. Let $y = \sum c_n x^n$ be nonzero series solution to $(x-3)y' + 2y = 0$. What is the radius of convergence of y ?
2. Find the recursion relation for a series solution centered at $x = 0$.

$$(x-1)y'' + (x-2)y' + y = 0$$

3. For each of the equations below determine all singular points (all other points are ordinary). For each singular point, $x = a$ do the following:
 - For each singular point, translate the equation using $t = x - a$ and put the equation in the form $t^2 y'' + tp(t)y' + q(t)y = 0$.
 - Determine $p(t)$ and $q(t)$. Determine if $x = a$ is a regular singular point or an irregular singular point.
 - If $x = a$ is a regular singular point, find p_0 and q_0 . Determine the exponents of the singular point.

(a) $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$.

(b) $xy'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$.

(c) $(x^2 - 1)y'' + y' - y = 0$.

(d) $(x^2 - 1)y'' + \frac{1}{x-1}y' - y = 0$.

(e) $(x^2 - 1)y'' + y' - \frac{1}{x+1}y = 0$.

(f) $y'' + \frac{1}{\sin \pi x}y' - y = 0$.

(g) $x^2(x^2 - 1)y'' + \frac{1}{\sin \pi x}y' - 2y = 0$.

4. x is population with independent variable t , time. Discuss stability and population of the models below.

(a) $x' = x^2 - 2x - 1$

(b) $x' = x^2 - 2x + 1$

(c) $x' = x^2 - 2x + 2$

(d) $x' = x^3 - 2x^2 + x$

(e) $x' = (x-4)^2(x-5)(6-x)$

5. A 200L tank contains a 5 g/L salt solution. A 20 g/L salt solution is pumped in at 10 L/min and the tanks is drained at 10 L/min. Find an expression for the smount of salt in the tank as a function of time.
6. Find a power series solution to $xy'' + 3xy' - y = 0$ centered at $x = 1$. (Just worry about finding the recursion relation and the first several terms).

7. Compute the convolution $(\sin 5t) * (\cos 5t)$.

8. Solve

$$\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

9. Convert the equation, using the Laplace transform, to a differential equation in $\mathcal{L}\{x\} = X$.

$$tx'' + x' + tx = 0, \quad x(0) = 1, x'(0) = 0$$

10. Solve $ye^{xy} dx + xe^{xy} dy = 0$

11. Find $\mathcal{L}^{-1}\{1/\sqrt{s-1}\} =$

12. Determine the value of m that will yield resonance

$$mx'' + x = 10 \cos 3t$$

13. The equation

$$y'' - \frac{2}{x}y' + \frac{2}{x^2} = \sqrt{x}$$

and the complementary solution $y_c = c_1x + c_2x^2$, find a particular solution.

14. Find a general solution to $y' = 1 + 2x + y + 2xy$. Hint: separable.

15. Solve $y'' + 2y' + y = 0$, $y(0) = 5$, $y'(0) = -3$.

16. Match the slope fields and the differential equations (the ranges for each graph are all $-2 \leq x, y \leq 2$).

$$y' = x + 2y, \quad y' = x - 2y, \quad y' = x + 4y, \quad y' = x - 4y,$$

