Math 217 - December 7, 2010
Solutions

---

1. Solve the initial value problems

   (a) $x^2y' + xy = 0, y(1) = 2$. **Solution:** $y = 2/x$

   (b) $x^2y' + xy = 2, y(1) = 2$. **Solution:** $y = 2(1 + \ln x)/x$

   (c) $x^2y' + xy = 1/x, y(1) = 2$. **Solution:** $y = (3x - 1)/x^2$

   (d) $y' + y/(1 - x) = 3, y(2) = 2$. **Solution:** $y = 3(x - 1)\ln(x - 1) + 2(x - 1)$

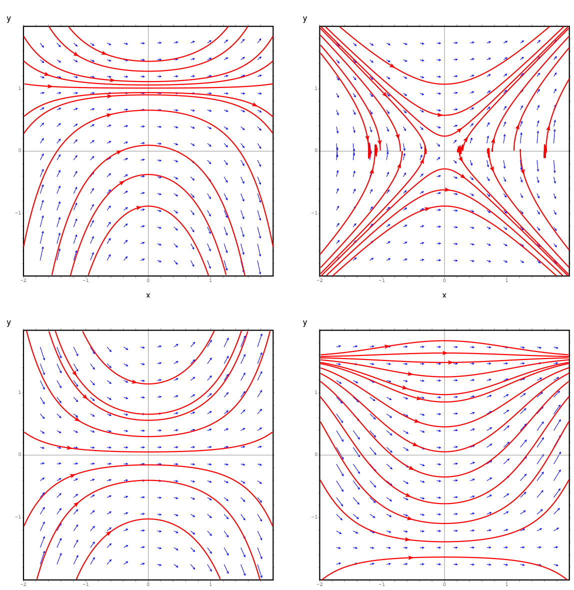
2. Discuss the equilibrium solutions and stability for the equations

   (a) $x' = (2x - 3)(x - 4)^2(x + 7)^3$

   (b) $x' = (x - 4)(x^2 - 4)$

3. Match the slope fields and the differential equations (which one does't match up?).

$$y' = xy$$
$$y' = x - y$$
$$y' = x/y$$
$$y' = x\cos y$$
$$y' = x(y - 1)$$



4. Use Euler's method to approximate solutions

   (a) $y' = x + y, y(0) = 2$. Using $h = 0.5$ approximate $y(1)$. **Solution:** $y(1) \approx 4.75$

   (b) $y' = x^2 + y, y(1) = 2$. Using $h = 0.5$ approximate $y(2)$. **Solution:** $y(2) \approx 6.375$

(c) $y' = x - y, y(1) = 2$. Using $h = 0.5$ approximate $y(0)$. **Solution:** $y(0) \approx 3.5$

5. Find all ordinary and singular points of the given equations. Determine if each singular point is regular or irregular. For the regular singular points, find p_0, q_0 and the exponents. What does our theorem say about Frobenius solutions at the regular singular points?

   (a) $y'' + y' + y = 0$. **Solution:** All points are ordinary.

   (b) $x^2y'' + y' + y = 0$. **Solution:** $x = 0$ is the only singular point, it is irregular.

   (c) $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$. **Solution:** $x = 0$ is the only singular point, it is regular with exponents $1, -1$.

   (d) $x^2y'' + (1 - \cos x)y' - y = 0$. **Solution:** $x = 0$ is the only singular point, it is regular with exponents $(1 \pm \sqrt{t})/2$.

   (e) $(x^2 - x)y'' + (\sin x)y' - (\cos x)y = 0$. **Solution:** Singular points $x = 0, 1$. $x = 0$ is regular with exponents $0, 1$. $x = 1$ is regular with exponents (how do you find these??).