Math 217 - December 6, 2010

Theorem 1 (Frobenius Series). *Suppose $x = 0$ is a regular singular point of the equation $x^2y'' + xp(x)y' + q(x)y = 0$. Let r_1 and r_2 be the roots, $r_1 \geq r_2$ of the indicial equation $r(r-1) + p_0 r + q_0 = 0$. Then*

1. *There exists a solution to the differential equation of the form $y_1 = x^{r_1} \sum_{n=0}^{\infty} a_n x^2$.*
2. *If $r_1 - r_2 \notin \mathbb{Z}$, there there exists a second linearly independent solution of the form $y_1 = x^{r_2} \sum_{n=0}^{\infty} b_n x^2$.*
3. *If $r_1 - r_2 \in \mathbb{Z}$ then a second linearly independent solution can be found using methods in section 8.4.*

1. What does the above theorem say about solutions to the differential equation

$$(\sin x)y'' + \frac{1}{x}y' + y = 0$$

Lecture Problems

2. Using the root $r = -1$, find the Frobenius series solution to

$$2x^2y'' + 3xy' - (x^2 + 1)y = 0$$

3. Find a Frobenius solution to

$$xy'' + e^x y = 0$$

4. Find a Frobenius solution to

$$4x^2y'' - 4x^2y' + (1 + 2x)y = 0$$