Math 217 - December 3, 2010
Solutions

1. Let $p_0 = 1$ and $q_0 = -1$.
   (a) Solve the equation for r: $r(r-1) + p_0 r + q_0 = 0$. **Solution:** $r = -1, 1$
   (b) For each solution, verify that x^r is a solution to
   $$x^2 y'' + x p_0 y' + q_0 y = 0$$
2. Let $p_0 = 1$ and $q_0 = -9$.
   (a) Solve the equation for r: $r(r-1) + p_0 r + q_0 = 0$. **Solution:** $r = -3, 3$
   (b) For each solution, verify that x^r is a solution to
   $$x^2 y'' + x p_0 y' + q_0 y = 0$$
3. Let $p_o = 3$ and $q_0 = -8$.
   (a) Solve the equation for r: $r(r-1) + p_0 r + q_0 = 0$. **Solution:** $r = -4, 2$
   (b) For each solution, verify that x^r is a solution to
   $$x^2 y'' + x p_0 y' + q_0 y = 0$$
4. Let $p_0 = 3/2$ and $q_0 = -1/2$.
   (a) Solve the equation for r: $r(r-1) + p_0 r + q_0 = 0$. **Solution:** $r = -1, 1/2$
   (b) For each solution, verify that x^r is a solution to
   $$x^2 y'' + x p_0 y' + q_0 y = 0$$

Lecture Problems

5. Determine if $x = 0$ is an ordinary point, a regular singular point or an irregular singular point for the equations. If the singular point is regular, find p_0, q_0 and the exponents.
   (a) (Bessl's equation) $x^2 y'' + x y' + (x^2 - n^2) y = 0$
   Solution: Regular, $p_0 = 1, q_0 = -n^2$, roots: $-n, n$.
   (b) $x^2 y'' + y' + y = 0$.
   Solution: In this case $p = \frac{1}{x}$, a clear problem. Irregular.
   (c) $(x \sin x) y'' + (\cos x) y' + e^x y = 0$
   Solution: This becomes
   $$x^2 y'' + \frac{x \cos x}{\sin x} + x e^x y = 0$$
   $p_0 = 1, q_0 = 0$, root: 0.

(d) $2x^2y'' + 3xy' - (x^2+1)y = 0$

Solution: Regular with $p_0 = 3/2, q_0 = -1/2$. Roots: $-1, 1/2$.

6. Using the root $r = -1$, find the Frobenius series solution to

$$2x^2y'' + 3xy' - (x^2+1)y = 0$$

Solution:

$$y = b_0 x^{-1}\left(1 + \frac{1}{2}x^2 + \frac{1}{40}x^4 + \frac{1}{2160}x^6 + \frac{1}{224640}x^8 + \cdots\right)$$