

Math 217 - December 3, 2010

1. Let $p_0 = 1$ and $q_0 = -1$.

(a) Solve the equation for r : $r(r - 1) + p_0 r + q_0 = 0$.

(b) For each solution, verify that x^r is a solution to

$$x^2 y'' + x p_0 y' + q_0 y = 0$$

2. Let $p_0 = 1$ and $q_0 = -9$.

(a) Solve the equation for r : $r(r - 1) + p_0 r + q_0 = 0$.

(b) For each solution, verify that x^r is a solution to

$$x^2 y'' + x p_0 y' + q_0 y = 0$$

3. Let $p_0 = 3$ and $q_0 = -8$.

(a) Solve the equation for r : $r(r - 1) + p_0 r + q_0 = 0$.

(b) For each solution, verify that x^r is a solution to

$$x^2 y'' + x p_0 y' + q_0 y = 0$$

4. Let $p_0 = 3/2$ and $q_0 = -1/2$.

(a) Solve the equation for r : $r(r - 1) + p_0 r + q_0 = 0$.

(b) For each solution, verify that x^r is a solution to

$$x^2 y'' + x p_0 y' + q_0 y = 0$$

1. Let $p_0 = 1$ and $q_0 = -1$.

(a) Solve the equation for r : $r(r-1) + p_0r + q_0 = 0$. **Solution:**
 $r = -1, 1$

(b) For each solution, verify that x^r is a solution to

$$x^2y'' + xp_0y' + q_0y = 0$$

2. Let $p_0 = 1$ and $q_0 = -9$.

(a) Solve the equation for r : $r(r-1) + p_0r + q_0 = 0$. **Solution:**
 $r = -3, 3$

(b) For each solution, verify that x^r is a solution to

$$x^2y'' + xp_0y' + q_0y = 0$$

3. Let $p_0 = 3$ and $q_0 = -8$.

(a) Solve the equation for r : $r(r-1) + p_0r + q_0 = 0$. **Solution:**
 $r = -4, 2$

(b) For each solution, verify that x^r is a solution to

$$x^2y'' + xp_0y' + q_0y = 0$$

4. Let $p_0 = 3/2$ and $q_0 = -1/2$.

(a) Solve the equation for r : $r(r-1) + p_0r + q_0 = 0$. **Solution:**
 $r = -1, 1/2$

(b) For each solution, verify that x^r is a solution to

$$x^2y'' + xp_0y' + q_0y = 0$$

Lecture Problems

5. Determine if $x = 0$ is an ordinary point, a regular singular point or an irregular singular point for the equations. If the singular point is regular, find p_0 , q_0 and the exponents.

(a) (Bessl's equation) $x^2y'' + xy' + (x^2 - n^2)y = 0$

(b) $x^2y'' + y' + y = 0$.

(c) $(x \sin x)y'' + (\cos x)y' + e^x y = 0$

(d) $2x^2y'' + 3xy' - (x^2 + 1)y = 0$

Lecture Problems

5. Determine if $x = 0$ is an ordinary point, a regular singular point or an irregular singular point for the equations. If the singular point is regular, find p_0 , q_0 and the exponents.

(a) (Bessl's equation) $x^2 y'' + xy' + (x^2 - n^2)y = 0$

Solution: Regular, $p_0 = 1$, $q_0 = -n^2$, roots: $-n, n$.

(b) $x^2 y'' + y' + y = 0$.

Solution: In this case $p = \frac{1}{x}$, a clear problem. Irregular.

(c) $(x \sin x)y'' + (\cos x)y' + e^x y = 0$

Solution: This becomes

$$x^2 y'' + \frac{x \cos x}{\sin x} + x e^x y = 0$$

$p_0 = 1$, $q_0 = 0$, root: 0.

(d) $2x^2 y'' + 3xy' - (x^2 + 1)y = 0$

Solution: Regular with $p_0 = 3/2$, $q_0 = -1/2$. Roots: $-1, 1/2$.

6. Using the root $r = -1$, find the Frobenius series solution to

$$2x^2y'' + 3xy' - (x^2 + 1)y = 0$$

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Solution:

$$y = b_0x^{-1} \left(1 + \frac{1}{2}x^2 + \frac{1}{40}x^4 + \frac{1}{2160}x^6 + \frac{1}{224640}x^8 + \cdots \right)$$