Math 217 - December 2, 2010

1. Find the first several (≈ 5) terms in the recurrance relation. Find the two linearly independent solutions.

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = \frac{c_n + c_{n-1}}{(n+2)(n+1)}$$

1. Find the first several (≈ 5) terms in the recurrance relation. Find the two linearly independent solutions.

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = \frac{c_n + c_{n-1}}{(n+2)(n+1)}$$

Solution:

$$\begin{aligned} y = & c_0 \left(1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{30}x^5 + \cdots\right) \\ & + c_1 \left(x + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{120}x^5 + \cdots\right) \end{aligned}$$

Lecture Problems

2. Given the recurrance relation, find the solution to the DE

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = c_n - 2c_{n-1}$$

3. Given the recurrance relation, find the solution to the DE

$$c_2 = c_3 = c_4 = c_5 = 0, \quad c_{n+2} = \frac{c_{n-4} + n^2 c_{n+1}}{(n+1)(n+2)}$$

Lecture Problems

2. Given the recurrance relation, find the solution to the DE

$$c_2 = \frac{1}{2}c_0, \quad c_{n+2} = c_n - 2c_{n-1}$$

Solution:

$$\begin{aligned} y =& c_0\left(1 + \frac{1}{2}x^2 - 2x^3 + \frac{1}{2}x^4 - 3x^5 + \frac{9}{2}x^6 - 4x^7 + \frac{21}{2}x^8 + -13x^9 \right. \\ & + c_1\left(x + x^3 - 2x^4 + x^5 - 4x^6 + 5x^7 - 6x^8 + 13x^9 + \cdots\right) \end{aligned}$$

3. Given the recurrance relation, find the solution to the DE

$$c_2 = c_3 = c_4 = c_5 = 0, \quad c_{n+2} = \frac{c_{n-4} + n^2 c_{n+1}}{(n+1)(n+2)}$$

Solution:

$$\begin{aligned} y =& c_0\left(1 + \frac{1}{30}x^6 + \frac{5}{252}x^7 + \frac{5}{392}x^8 + \frac{5}{576}x^9 + \cdots\right) \\ & + c_1\left(x + \frac{1}{42}x^7 + \frac{3}{196}x^8 + \frac{1}{96}x^9 + \frac{1}{135}x^{10} + \cdots\right) \end{aligned}$$

4. Given the recurrance relation, find the solution to the DE

$$c_1 = \frac{1}{2}c_0, c_2 = 0, \quad c_{n+2} = \frac{2c_n - 3c_{n-1}}{n^2+1}$$

5. Given the recurrance relation, find the solution to the DE

$$c_0 = 1, c_2 = -3c_1, c_3 = 0, \quad c_{n+2} = \frac{2c_{n+1} - 3c_n + c_{n-1}}{n+1}$$

4. Given the recurrance relation, find the solution to the DE

$$c_1 = \frac{1}{2}c_0, c_2 = 0, \quad c_{n+2} = \frac{2c_n - 3c_{n-1}}{n^2+1}$$

Solution:

$$\begin{aligned} y =& c_0\left(1 + \frac{1}{2}x - \frac{3}{5}x^4 - \frac{3}{20}x^5 - \frac{6}{85}x^6 - \frac{3}{260}x^7 + \cdots\right) \\ &+ c_3\left(x^3 + \frac{1}{5}x^5 - \frac{1}{10}x^7 - \frac{2}{125}x^9 + \cdots\right) \end{aligned}$$

5. Given the recurrance relation, find the solution to the DE

$$c_0 = 1, c_2 = -3c_1, c_3 = 0, \quad c_{n+2} = \frac{2c_{n+1} - 3c_n + c_{n-1}}{n+1}$$

Solution:

$$\begin{aligned} y =& 1 + c_1\left(x - 3x^2 + \frac{10}{3}x^4 + \frac{11}{12}x^5 - \frac{49}{30}x^6 - \frac{161}{360}x^7 + \cdots\right) \\ &+ c_3\left(x^3 + \frac{2}{3}x^4 - \frac{5}{12}x^5 - \frac{11}{30}x^6 + \frac{71}{360}x^7 + \cdots\right) \end{aligned}$$

6. Find a series solution to the DE

$$y'' - e^x y = 0$$

7. Find a series solution to the DE

$$y'' + (\sin x)y = 0$$

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$$y'' - e^x y = 0$$

Solution:

$$y = c_0 \left(1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{40}x^5 + \frac{1}{80}x^6 + \cdots\right)$$
$$+ c_1 \left(x - \frac{1}{6}x^3 - \frac{1}{12}x^4 - \frac{1}{60}x^5 + \frac{1}{360}x^6 + \cdots\right)$$

7. Find a series solution to the DE

$$y'' + (\sin x)y = 0$$

Solution:

$$y = c_0 \left(1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 - \frac{1}{5040}x^7 - \cdots\right)$$
$$+ c_1 \left(x - \frac{1}{12}x^4 + \frac{180^{6}}{x} + \frac{1}{504}x^7 - \cdots\right)$$

8. Find a series solution to the DE

$$y'' - e^x y = \sin x$$

9. Find a series solution to the DE

$$y'' + (\cos x)y' - (\sin x)y = e^x$$

8. Find a series solution to the DE

$$y'' - e^x y = \sin x$$

Solution:

$$y = c_0 \left(1 - \frac{1}{2}x^2 + \frac{1}{120}x^5 + \frac{1}{144}x^6 + \frac{1}{560}x^7 + \cdots \right) + c_1 \left(x - \frac{1}{12}x^4 - \frac{1}{30}x^5 - \frac{1}{360}x^6 + + \frac{1}{504}x^7 + \cdots \right)$$

9. Find a series solution to the DE

$$y'' + (\cos x)y' - (\sin x)y = e^x$$

Solution:

$$y = c_0 \left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 - \frac{1}{720}x^6 + \frac{1}{315}x^7 + \cdots \right) + c_1 \left(x + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{60}x^5 + \frac{1}{180}x^6 - \frac{1}{630}x^7 + \cdots \right)$$