Math 132: Discussion Session: Week 7

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn this paper in at the end of the class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Scoring: Correct Answers Grade
0-3 0%
4-6 80%
7-11 100%

Group Members:

1.) \( \int e^{4x} \sin 2x \, dx = \)

2.) \( \int_0^{\pi/4} \cos (3x) \cos x \, dx = \)

3.) \( \int \sin^3 x \cos^2 x \, dx = \)

4.) \( \int x \sin^3 x \cos^2 x \, dx = \)

5a.) The expression is

5b.) The number of people infected is

5c.) The average number of people per day is

6a.) The set-up is

6b.) The volume is

7a.) The set-up is

7b.) The volume is
Math 132: Discussion Session: Week 7 Problem Sheet

Directions: In groups of 3-4 students, work the problems on this page.

Section 7.1 and 7.2: Integrate/Evaluate the following:

1.) \( \int e^{4x} \sin 2x \, dx \)

2.) \( \int_{0}^{\pi/4} \cos 3x \cos x \, dx \)

3.) \( \int \sin^3 x \cos^2 x \, dx \)

4.) \( \int x \sin^3 x \cos^2 x \, dx \)

Review:

5.) A new virus has just been declared an epidemic by health officials. Currently, 10,000 people have the disease, and it is estimated that \( t \) days from now, new cases will be reported at the rate of \( R(t) = 10te^{-t/10} \) people per day.
   a.) Set up the expression needed to determine how many people will be infected 30 days from now. (Hint: This expression should involve an integral.)
b.) Evaluate your expression above to determine how many people will be infected 30 days from now.

c.) What was the average number of new cases per day during the 30-day period?

6.) We want to obtain the volume when the area between $y = \sin x$ and $y = \cos x$ on $\left[ 0, \frac{\pi}{4} \right]$ is rotated about the $x$-axis.
   a.) Using the disk/washer method, set up the integral which would give the volume.

   b.) Evaluate the integral to find the volume.

7.) Take the area under the function $y = \sin^3 x \cos^2 x$ on $\left[ 0, \frac{\pi}{2} \right]$ and rotate it about the $y$-axis.

   a.) Set up the integral needed to find the resulting volume.

   b.) Find the volume.
Math 132: Discussion Session: Week 7 Problem Sheet

**Directions:** In groups of 3-4 students, work the problems on this page.

**Section 7.1 and 7.2:** Integrate the following:

1.) \( \int e^{4x} \sin 2x \, dx \)
   
   \[ u = e^{4x} \quad dv = \sin 2x \, dx \]
   
   \[ du = 4e^{4x} \, dx \quad v = -\frac{1}{2} \cos 2x \]
   
   \[ \int e^{4x} \sin 2x \, dx = -\frac{1}{2} e^{4x} \cos 2x + \int 2e^{4x} \cos 2x \, dx \]
   
   \[ u = e^{4x} \quad dv = 2 \cos 2x \, dx \]
   
   \[ du = 4e^{4x} \, dx \quad v = \sin 2x \]
   
   \[ \int e^{4x} \sin 2x \, dx = -\frac{1}{2} e^{4x} \cos 2x + e^{4x} \sin 2x - 4 \int e^{4x} \sin 2x \, dx \]
   
   \[ 5 \int e^{4x} \sin 2x \, dx = -\frac{1}{2} e^{4x} \cos 2x + e^{4x} \sin 2x \]
   
   \[ \int e^{4x} \sin 2x \, dx = -\frac{1}{10} e^{4x} \cos 2x + \frac{1}{5} e^{4x} \sin 2x + c \]

2.) \( \int_{0}^{\pi/4} \cos (3x) \cos x \, dx \)
   
   \[ \int_{0}^{\pi/4} \left[ \frac{1}{2} \cos (4x) + \frac{1}{2} \cos (2x) \right] \, dx = \left( \frac{1}{8} \sin (4x) + \frac{1}{4} \sin (2x) \right) \bigg|_{0}^{\pi/4} = \frac{1}{4} \]

3.) \( \int \sin^3 x \cos^2 x \, dx \)
   
   \[ = \int \sin x (1 - \cos^2 x) \cos^2 x \, dx = \int (\cos^2 x - \cos^4 x) \sin x \, dx \]
   
   let \( u = \cos x \)
   
   then \( du = -\sin x \, dx \)
   
   \[ = \int -u^2 + u^4 \, du = -\left( \frac{1}{3} u^3 - \frac{1}{5} u^5 \right) + c = -\left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + c \]

4.) \( \int x \sin^3 x \cos^2 x \, dx \)
   
   \[ u = x \quad dv = \sin^3 x \cos^2 x \, dx \]
   
   \[ du = dx \quad v = -\left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) \]
   
   based on the answer to the previous problem
   
   \[ \int x \sin^3 x \cos^2 x \, dx = -x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \int \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) \, dx \]
\[
-x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{1}{3} \int \cos x \left( 1 - \sin^2 x \right) dx - \frac{1}{5} \int \cos x \left( 1 - 2 \sin^2 x + \sin^4 x \right) dx
\]

\[
u = \sin x
\]

\[
du = \cos x \, dx
\]

\[
-x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{1}{3} \left[ (1 - u^2) du - \frac{1}{5} \left( 1 - 2u^2 + u^4 \right) du \right]
\]

\[
-x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{1}{3} \left( u - \frac{1}{3} u^3 \right) - \frac{1}{5} \left( u - \frac{2}{3} u^3 + \frac{1}{5} u^5 \right) + c
\]

\[
-x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{1}{3} \left( \sin x - \frac{1}{3} \sin^3 x \right) - \frac{1}{5} \left( \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x \right) + c
\]

\[
-x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{2}{15} \sin x + \frac{1}{45} \sin^3 x - \frac{1}{25} \sin^5 x + c
\]

Review:

5.) A new virus has just been declared an epidemic by health officials. Currently, 10,000 people have the disease, and it is estimated that \( t \) days from now, new cases will be reported at the rate of \( R(t) = 10e^{-t/10} \) people per day.

a.) Set up the expression needed to determine how many people will be infected 30 days from now. (Hint: This expression should involve an integral.)

\[
10,000 + \int_0^{30} 10te^{-t/10} \, dt
\]

b.) Evaluate your expression above to determine how many people will be infected 30 days from now.

\[
10,000 + \int_0^{30} 10te^{-t/10} \, dt
\]

\[
u = 10t \quad dv = e^{-t/10} \, dt
\]

\[
du = 10 \, dt \quad v = -10e^{-t/10}
\]

\[
10,000 \left[ -100e^{-t/10} \right]_0^{30} + \int_0^{30} 1000e^{-t/10} \, dt = 10,000 + \left( -100e^{-3/10} - 1000e^{-3/10} \right)_0^{30}
\]

\[
10,000 + (1000 - 4000e^{-3}) = 11,000 - 4000e^{-3}
\]

c.) What was the average number of new cases per day during the 30-day period?

\[
\frac{1}{30} \int_0^{30} 10te^{-t/10} \, dt
\]

\[
\frac{1}{30} \left( -100e^{-t/10} - 1000e^{-t/10} \right)_0^{30}
\]

\[
\frac{1}{30} (1000 - 4000e^{-3})
\]
6.) We want to obtain the volume when the area between \( y = \sin x \) and \( y = \cos x \) on \( [0, \frac{\pi}{4}] \) is rotated about the \( x \)-axis.

a.) Using the disk/washer method, set up the integral which would give the volume.
\[
R = y_{\text{top}} = \cos x
\]
\[
r = y_{\text{bottom}} = \sin x
\]
\[
h = dx
\]
\[
\pi \int_0^{\pi/4} \left( \cos^2 x - \sin^2 x \right) dx
\]

b.) Evaluate the integral to find the volume.
\[
\pi \int_0^{\pi/4} \cos 2x \, dx = \frac{\pi}{2} \sin 2x \bigg|_0^{\pi/4} = \frac{\pi}{2}
\]

7.) Take the area under the function \( y = \sin^3 x \cos^2 x \) on \( \left[0, \frac{\pi}{2}\right] \) and rotate it about the \( y \)-axis.

a.) Set up the integral needed to find the resulting volume.

Shells is easiest.
\[
r = x
\]
\[
h = y = \sin^3 x \cos^2 x
\]
\[
\Delta r = dx
\]
\[
2\pi \int_0^{\pi/2} x \sin^3 x \cos^2 x \, dx
\]

b.) Find the volume.

From Question 4, the antiderivative was already found…
\[
2\pi \left[ -x \left( \frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x \right) + \frac{2}{15} \sin x + \frac{1}{45} \sin^3 x - \frac{1}{25} \sin^5 x \right]_0^{\pi/2}
\]
\[
2\pi \left( -\frac{\pi}{2} (0) + \frac{2}{15} + \frac{1}{45} - \frac{1}{25} \right) - (0) = \frac{52\pi}{225}
\]