Math 132: Discussion Session: Week 6

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn this paper in at the end of the class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Scoring: Correct Answers Grade
0-4 0%
5-7 80%
8-13 100%

Group Members:

1.) The average is:

2a.) $c =$
2b.) $c =$

3a.) $\int \arctan(2x)dx =$
3b.) $\int \cos x \cosh x dx =$

4a.i.) The integral is:

4a.ii.) The volume is:

4b.i.) The integral is:

4b.ii.) The volume is:

5a.) The integral is:

5b.) The volume is:

6a.) The integral is:

6b.) The volume is:
Math 132: Discussion Session: Week 6 Problem Sheet

**Directions:** In groups of 3-4 students, work the problems on this page.

**Section 6.5:**
1.) A patient is injected with a drug, and $t$ hours later, the concentration of the drug remaining in the patient’s blood stream is given by $C(t) = \frac{3t}{(t^2 + 36)^{3/2}}$ mg/cm$^3$. What is the average concentration of the drug during the first 8 hours after the injection?

2.) Find the value(s) of $c$ which satisfy the MVT for integrals given the specified function and the specified interval:
   a.) $f(x) = -3(2x-6)^{1/2}$ on $[3,5]$ 
   b.) $f(x) = \frac{4}{(2x+6)^2}$ on $[-6,-5]$

**Section 7.1:**
3.) Integrate the following:
   a.) $\int \arctan(2x)dx$

   b.) $\int \cos x \cosh x dx$

   (Hint: $\frac{d}{dx} \cosh x = \sinh x$ and $\frac{d}{dx} \sinh x = \cosh x$)
4.) Take the area bounded by $y = 5x^3$, $x = 0$, $y = 0$, and $x = 2$ and find the volume generated when this area is…
   a.) Rotated about the line $x = 4$
      i.) Set up the integral.
      ii.) Find the volume.

   b.) Rotated about the line $y = -2$
      i.) Set up the integral.
      ii.) Find the volume.

5.) The area under $y = \sin\left(\frac{\pi x}{2}\right)$ on $[0,2]$ is rotated about the y-axis. Find the volume generated.
   a.) Set up the integral.
   b.) Find the volume.

6.) Find the volume generated when the area under $f(x) = xe^x + 2$ between $x = -3$ and $x = 1$ is rotated about the line $x = 1$. (The graph of $f$ is shown to the right.)
   a.) Set up the integral.
   b.) Find the volume.
Directions: In groups of 3-4 students, work the problems on this page.

Section 6.5:

1.) A patient is injected with a drug, and \( t \) hours later, the concentration of the drug remaining in the patient’s bloodstream is given by \( C(t) = \frac{3t}{(t^2 + 36)^{3/2}} \) mg/cm\(^3\). What is the average concentration of the drug during the first 8 hours after the injection?

\[
C_{ave} = \frac{1}{8-0} \int_0^8 \frac{3t}{(t^2 + 36)^{3/2}} dt
\]

\[
u = t^2 + 36, \quad du = 2tdt
\]

\[
\int_0^{100} \int_{36}^{1/2} \frac{3}{u^{3/2}} du = \frac{3}{16} \cdot \frac{-2u^{-1/2}}{100}
\]

\[
= -\frac{3}{8} \left( \frac{1}{10} - \frac{1}{6} \right) = \frac{1}{40} \text{ mg/cm}^3
\]

2.) Find the value(s) of \( c \) which satisfy the MVT for integrals given the specified function and the specified interval:

b.) \( f(x) = -3(2x - 6)^{1/2} \) on \([3,5]\)  
b.) \( f(x) = \frac{4}{(2x + 6)^2} \) on \([-6,-5]\)

Both of these functions are continuous on their respective intervals...

\[
f_{ave} = \frac{1}{2} \int_3^5 -3(2x - 6)^{1/2} dx
\]

\[
u = 2x - 6, \quad du = 2dx
\]

\[
= \frac{1}{2} \int_0^4 -\frac{3}{2} u^{1/2} du
\]

\[
= -\frac{1}{2} u^{3/2} \bigg|_0^4 = -\frac{1}{2} (8 - 0) = -4
\]

\[
-3(2c - 6)^{1/2} = -4
\]

\[
(2c - 6)^{1/2} = \frac{4}{3}
\]

\[
2c - 6 = \frac{16}{9}
\]

\[
c = \frac{35}{9}
\]
Section 7.1:

3.) Integrate the following:

a.) \( \int \arctan(2x) \, dx \)
\[
\begin{align*}
  u &= \arctan(2x) & dv &= dx \\
  du &= \frac{2 \, dx}{1 + 4x^2} & v &= x \\
  x \arctan(2x) - \int \frac{2x}{1 + 4x^2} \, dx \\
  u &= 1 + 4x^2 \\
  du &= 8x \, dx \\
  x \arctan(2x) &= \frac{1}{4} \int \frac{du}{u} \\
  x \arctan(2x) &= \frac{1}{4} \ln |u| + c \\
  x \arctan(2x) &= \frac{1}{4} \ln |1 + 4x^2| + c
\end{align*}
\]

b.) \( \int \cos x \cosh x \, dx \)
\[
\begin{align*}
  u &= \cos x & dv &= \cosh x \, dx \\
  du &= -\sin x \, dx & v &= \sinh x \\
  \int \cos x \cosh x \, dx &= \cos x \sinh x + \int \sin x \sinh x \, dx \\
  u &= \sin x & dv &= \sinh x \, dx \\
  du &= \cos x \, dx & v &= \cosh x \\
  \int \cos x \cosh x \, dx &= \cos x \sinh x + \sin x \cosh x - \int \cos x \cosh x \, dx \\
  2 \int \cos x \cosh x \, dx &= \cos x \sinh x + \sin x \cosh x \\
  \int \cos x \cosh x \, dx &= \frac{\cos x \sinh x + \sin x \cosh x}{2} + c
\end{align*}
\]

Section 6.2, 6.3, 7.1:

4.) Take the area bounded by \( y = 5x^3, \ x = 0, \ y = 0, \) and \( x = 2 \) and find the volume generated when this area is…

a.) Rotated about the line \( x = 4 \)
   i.) Set up the integral.

Shells is easiest…
\[
\begin{align*}
  r &= 4 - x \\
  h &= y = 5x^3 \\
  V &= 2\pi \int_0^2 (4 - x)(5x^3) \, dx \\
  \Delta r &= dx
\end{align*}
\]
ii.) Find the volume.
\[
V = 2\pi \int_0^2 (4-x)(5x^3)\,dx = 2\pi \int_0^2 (20x^3 - 5x^4)\,dx \\
= 2\pi \left(5x^4 - x^5\right)_0^2 = 2\pi (80 - 32) = 96\pi
\]

b.) Rotated about the line \( y = -2 \)

i.) Set up the integral.

Washers is easiest…
\[
R = 5x^3 + 2 \\
r = 2 \\
h = dx \\
V = \pi \int_0^2 [(5x^3 + 2)^2 - 4]\,dx
\]

ii.) Find the volume.
\[
V = \pi \int_0^2 [(5x^3 + 2)^2 - 4]\,dx = \pi \int_0^2 (25x^6 + 20x^3)\,dx \\
= \pi \left(\frac{25}{7}x^7 + 5x^4\right)_0^2 = \pi \left(\frac{3200}{7} + 80\right) = \frac{3760\pi}{7}
\]

5.) The area under \( y = \sin\left(\frac{\pi x}{2}\right) \) on \([0,2]\) is rotated about the \( y \)-axis. Find the volume generated.

a.) Set up the integral.

Shells is easiest:
\[
r = x \\
h = \sin\left(\frac{\pi x}{2}\right) \\
V = 2\pi \int_0^2 x \sin\left(\frac{\pi x}{2}\right)\,dx
\]

b.) Find the volume.

This needs to be done by parts…
\[
u = x \quad dv = \sin\left(\frac{\pi x}{2}\right)\,dx \\
u = -\frac{2}{\pi} \cos\left(\frac{\pi x}{2}\right) \\
v = -\frac{2}{\pi} \cos\left(\frac{\pi x}{2}\right)
\]
\[
V = 2\pi \left[ -\frac{2x}{2\pi} \cos\left(\frac{\pi x}{2}\right) \right]_0^2 + \int_0^2 \frac{2}{2\pi} \cos\left(\frac{\pi x}{2}\right)\,dx \\
= 2\pi \left[ -\frac{2x}{2\pi} \cos\left(\frac{\pi x}{2}\right) \right]_0^2 + \frac{4}{\pi^2} \sin\left(\frac{\pi x}{2}\right)_0^2 \\
= 2\pi \left[ \left(\frac{4}{\pi}\right) + (0) \right] = 8
6.) Find the volume generated when the area under
\[ f(x) = xe^x + 2 \] between \( x = -3 \) and \( x = 1 \) is
rotated about the line \( x = 1 \). (The graph of \( f \) is
shown to the right.)
a.) Set up the integral.

Shells is easiest:
\[ r = 1 - x \]
\[ h = xe^x + 2 \]
\[ \Delta r = dx \]
\[ V = 2\pi \int_{-3}^{1} (1-x)(xe^x + 2) \, dx \]

b.) Find the volume.

\[
V = 2\pi \int_{-3}^{1} (1-x)(xe^x + 2) \, dx \\
= 2\pi \int_{-3}^{1} (xe^x + 2 - x^2e^x - 2x) \, dx \\
= 2\pi \int_{-3}^{1} (2 - 2x) \, dx + 2\pi \int_{-3}^{1} (x - x^2) \, e^x \, dx \\
\text{Needs to be done by parts twice} \\
u = x - x^2 \quad dv = e^x \, dx \\
du = (1 - 2x) \, dx \quad v = e^x \\
= 2\pi \left[ (2x - x^2) \bigg|_{-3}^{1} + (x - x^2) \bigg|_{-3}^{1} - \int_{-3}^{1} (1 - 2x) \, e^x \, dx \right] \\
u = 1 - 2x \quad dv = e^x \, dx \\
du = -2dx \quad v = e^x \\
= 2\pi \left[ (2x - x^2) \bigg|_{-3}^{1} + (x - x^2) \bigg|_{-3}^{1} - (1 - 2x) e^x \bigg|_{-3}^{1} - \int_{-3}^{1} 2e^x \, dx \right] \\
= 2\pi \left[ (1 + 15) + (0 + 12e^3) - (e - 7e^3) - (2e - 2e^3) \right] \\
= 2\pi \left[ 16 - e + 21e^3 \right]
\]
Note: Question 4 was intended to have used the function $y = 5x^3 + 2$ instead of $y = 5x^3$. In that case, the answers would have been:

a.) Rotated about the line $x = 4$
   i.) Set up the integral.
   Shells is easiest…
   
   $r = 4 - x$
   $h = y = 5x^3 + 2$ \( V = 2\pi \int_0^2 (4-x)(5x^3 + 2)\,dx \)
   \( \Delta r = dx \)

   ii.) Find the volume.
   
   \[
   V = 2\pi \int_0^2 (4-x)(5x^3 + 2)\,dx = 2\pi \int_0^2 (20x^3 + 8 - 5x^4 - 2x)\,dx = 2\pi \left( 5x^4 + 8x - x^5 - x^3 \right) \bigg|_0^2 = 2\pi (80 + 16 - 32 - 4) = 120\pi
   \]

b.) Rotated about the line $y = -2$
   i.) Set up the integral.
   Washers is easiest…
   
   $R = 5x^3 + 4$
   $r = 2$
   $h = dx$

   ii.) Find the volume.
   
   \[
   V = \pi \int_0^2 \left( (5x^3 + 4)^2 - 4 \right) dx = \pi \int_0^2 \left( 25x^6 + 40x^3 + 12 \right) dx = \pi \left( \frac{25}{7} x^7 + 10x^4 + 12x \right) \bigg|_0^2 = \pi \left( \frac{3200}{7} + 160 + 24 \right) = \frac{4488\pi}{7}
   \]