Math 132: Discussion Session: Week 5

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn this paper in at the end of the class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all the problems. Also, each group members should work through each problem, as similar problems may appear on the exam.

Scoring: Correct Answers Grade
0-3 0%
4-6 80%
7-10 100%

Group Members:

Section 6.2:
1a.) The equation is:

1b.) The integral is:

1c.) The volume is:

1d.) A Krispy Kreme volume is:

2.) The cost is $

3a.) The integral is:

3b.) The volume is:

4.) The volume is:

5a.) The volume is given by:

5b.) The volume is given by:
Math 132: Discussion Session: Week 5 Problem Sheet

Directions: In groups of 3-4 students, work the problems on this page.

Section 6.2:
1.) Our goal is to find the area of a donut. (This shape can be formed by taking a circle or radius $r$ whose center is a distance $R$ from the $y$-axis and rotating it about the $y$-axis.)

a.) If we consider the center of the donut to be at the origin, find an expression for the equation of the circle of radius $r$ which is to be rotated.

b.) Set-up the integral which will give the volume of the donut.

c.) Evaluate the integral, keeping in mind that $R$ and $r$ can be thought of as constants.

d.) After a little internet research, you find that Krispy Kreme donuts have dimensions as shown in the diagram. What is the volume of a Krispy Kreme donut?

2.) An engineer created a prototype for a solid brass doorknob by using the function

$$y = 0.5x^3 - 2x^2 + x - 0.5$$

and rotating it about the $x$-axis on the interval $[0, 3.5]$. (Both $x$ and $y$ have units of inches.) Brass has a density of 4.86 oz per cubic inch, and costs $0.10 per ounce. How expensive would it be to manufacture one doorknob? Give your final answer in terms of an integral. (Don’t attempt to evaluate the integral.)
3.) A ruptured pipe in an offshore oil rig produces a circular oil slick that is $y$ feet thick at a distance of $x$ feet from the rupture, where $y = \frac{3}{2+x}$. At the time the spill is contained, the radius of the oil slick is 7 feet. We wish to find the volume of oil that has been spilled. (Note the volume we want can be obtained by rotating the function about the y-axis.)

a.) Set up the integral needed to find the volume.

b.) What is the volume of oil that has been spilled?

4.) The base of a solid $R$ is formed from the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. Cross-sections of the solid, sliced parallel to the $x$-axis, are isosceles right triangles whose hypotenuse touches $R$. Find the volume of this solid.

5.) Suppose you have a region $S$ made from the area between $y = \log_2 x$ and $x - 3y = -1$. Set-up the integrals needed to find the volume if $S$ was rotated around:

a.) The line $y = -1$

b.) The line $x = 10$
Math 132: Discussion Session: Week 5 Problem Sheet

Directions: In groups of 3-4 students, work the problems on this page.

Section 6.2:
1.) Our goal is to find the area of a donut. (This shape can be formed by taking a circle or radius r whose center is a distance R from the y-axis and rotating it about the y-axis.)

a.) If we consider the center of the donut to be at the origin, find an expression for the equation of the circle of radius r which is to be rotated.

\[(x - R)^2 + y^2 = r^2\]

b.) Set-up the integral which will give the volume of the donut.

Solving for the right/left halves of this circle we see:

\[
\int_{-r}^{r} \pi \left[ (R + \sqrt{r^2 - y^2})^2 - (R - \sqrt{r^2 - y^2})^2 \right] dy = \pi \int_{-r}^{r} (4R\sqrt{r^2 - y^2}) dy = 4R\pi \int_{-r}^{r} (\sqrt{r^2 - y^2}) dy
\]

c.) Evaluate the integral, keeping in mind that R and r can be thought of as constants.
Notice that the integral can be thought of geometrically as half a circle.

\[4R\pi \cdot \frac{\pi r^2}{2} = 2Rr^2\pi^2\]

d.) After a little internet research, you find that Krispy Kreme donuts have dimensions as shown in the diagram. What is the volume of a Krispy Kreme donut?
Notice \( r = 0.7 \) inches and

\[ R = \frac{3.5}{2} - 0.7 = 1.05 \text{ inches} \]

\[ 2(1.05)(0.49)\pi^2 = 1.029\pi^2 \approx 10.156 \text{ in}^3 \]
2.) An engineer created a prototype for a solid brass doorknob by using the function 
\[ y = 0.5x^3 - 2x^2 + x - 0.5 \] 
and rotating it about the x-axis on the interval \([0, 3.5]\). (Both \(x\) and \(y\) have units of inches.) Brass has a density of 4.86 oz per cubic inch, and costs $0.10 per ounce. How expensive would it be to manufacture one doorknob? Give your final answer in terms of an integral. (Don’t attempt to evaluate the integral.)

\[
Volume = \pi \int_{0}^{3.5} (0.5x^3 - 2x^2 + x - 0.5)^2 \, dx \text{ in}^3
\]

\[
Cost = \pi \int_{0}^{3.5} (0.5x^3 - 2x^2 + x - 0.5)^2 \, dx \text{ in}^3 \times \frac{4.86 \text{ oz}}{\text{in}^3} \times \frac{$0.10}{\text{oz}}
\]

\[
= 0.486\pi \int_{0}^{3.5} (0.5x^3 - 2x^2 + x - 0.5)^2 \, dx
\]

3.) A ruptured pipe in an offshore oil rig produces a circular oil slick that is \(y\) feet thick at a distance of \(x\) feet from the rupture, where \(y = \frac{3}{2 + x}\). At the time the spill is contained, the radius of the oil slick is 7 feet. We wish to find the volume of oil that has been spilled. (Note the volume we want can be obtained by rotating the function about the \(y\)-axis.)

a.) Set up the integral needed to find the volume.

Notice that the equation solved for \(x\) is \(x = \frac{3}{y} - 2\) and the limits of integration are from \(x = 0\) to \(x = 7\), which corresponds to \(y = \frac{3}{2}\) and \(y = \frac{1}{3}\).

\[ V = \pi \int_{\frac{1}{3}}^{\frac{3}{2}} \left( \frac{3}{y} - 2 \right)^2 \, dy \]

b.) What is the volume of oil that has been spilled?

\[ V = \pi \int_{\frac{1}{3}}^{\frac{3}{2}} \left( \frac{3}{y} - 2 \right)^2 \, dy = \pi \int_{\frac{1}{3}}^{\frac{3}{2}} \left( \frac{9}{y^2} - \frac{12}{y} + 4 \right) \, dy = \pi \left[ - \frac{9}{y} - 12 \ln |y| + 4y \right]_{\frac{1}{3}}^{\frac{3}{2}}
\]

\[ = \pi \left( -6 - 12 \ln(3/2) + 6 + 27 + 12 \ln(1/3) - \frac{4}{3} \right) = \pi \left( \frac{77}{3} + 12 \ln \frac{2}{9} \right) \text{ ft}^3 \]
4.) The base of a solid $R$ is formed from the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. Cross-sections of the solid, sliced parallel to the $x$-axis, are isosceles right triangles whose hypotenuse touches $R$. Find the volume of this solid.

[Diagram showing a solid with cross-sections]

Volume of slice $= x^2 \, dy$

$x^2 = 9 - \frac{9}{16} y^2$

\[
\int_{-4}^{4} \left(9 - \frac{9}{16} y^2\right) \, dy = \left[9y - \frac{3}{16}y^3\right]_{-4}^{4} = (36 - 12) - (-36 + 12) = 48
\]

5.) Suppose you have a region $S$ made from the area between $y = \log_2 x$ and $x - 3y = -1$. Set-up the integrals needed to find the volume if $S$ was rotated around:

a.) The line $y = -1$

b.) The line $x = 10$

Note the points of intersection are $(2, 1)$ and $(8, 3)$

\[
\pi \int_{2}^{8} \left[(1 + \log_2 x)^2 - \left(\frac{x + 4}{3}\right)^2\right] \, dx \quad \pi \int_{1}^{3} \left[(10 - 2^y)^2 - (11 - 3y)^2\right] \, dx
\]