Section 7

6.1: Areas

1. **Solution:** Solve to get intersections at (0, 0) and (7, 56).

   \[
   A = \int_{0}^{7} (x^2 - x) - (4x^2 - 20x) \, dx = \frac{245}{2}
   \]

2. **Solution:** Solve to get intersections at (2, 18) and (5, 60).

   \[
   A = \int_{2}^{5} (x^2 + 7x) - (2x^2 + 10) \, dx = \frac{9}{2}
   \]

3. **Solution:** Solve \( y^2 - 4 = 2y - y^2 \) to get \((-3, -1)\) and \((0, 2)\).
For the area between $x = y^2 - 4$ and $x = 2y - y^2$, the key point is that you want to integrate with respect to $y$ and not $x$.

\[
\int_{-1}^{2} (x_{\text{big}} - x_{\text{small}}) \, dy = \int_{-1}^{2} (2y - 4) - (y^2 - 4) \, dy = 9
\]

This can also be done with a $dx$ integral, but it isn’t pretty. You have to solve for $y$, which is easy for one of the functions but a bit messy for the other (use quadratic formula):

\[
A = \int_{-4}^{-3} \sqrt{x + 4} - (-\sqrt{x + 4}) \, dx + \int_{-3}^{0} \sqrt{x + 4} - (1 - \sqrt{1 - x}) \, dx + \int_{0}^{1} (1 + \sqrt{1 - x}) - (1 - \sqrt{1 - x}) \, dx
\]

\[
= \frac{4}{3} + \frac{19}{3} + \frac{4}{3} = 9
\]

4. **Solution:** Points of intersection are $(1, -2)$ and $(4, 4)$. The tricky part is the $dx$ integral.

\[
A = \int_{-2}^{4} \left( \frac{y + 4}{2} - \frac{y^2}{4} \right) \, dy = 9
\]

\[
A = \int_{0}^{1} 2\sqrt{4x} \, dx + \int_{1}^{4} \left( \sqrt{4x} - (2x - 4) \right) \, dx = \frac{8}{3} + \frac{19}{3} = 9
\]

5. **Solution:** Intersection points are at $x = \pi/4$ and $x = 5\pi/4$. 

\[ A = \int_{0}^{2\pi} |\cos x - \sin x| \, dx \\
= \int_{0}^{\pi/4} (\cos x - \sin x) \, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) \, dx + \int_{5\pi/4}^{2\pi} (\cos x - \sin x) \, dx \\
= (\sqrt{2} - 1) + (2\sqrt{2}) + (\sqrt{2} + 1) = 4\sqrt{2} \]

**Solution:** Intersection points are at \( x = 0 \) and \( x = 1 \).

\[ A = \int_{0}^{1} \sqrt{x} - x^2 \, dx = \frac{1}{3} \]

**Solution:** It is helpful to graph the region:
8.

**Solution:** This is really important to graph:

\[
A = \int_{0}^{2} (x + 1 - xe^{-x^2}) \, dx = \frac{7}{2} + \frac{e^{-4}}{2}
\]

Graph and find the intersection points: \((-1, 12)\) and \((3, 28)\). Once you do this you can easily write down the area integral:

\[
A = \int_{-2}^{-1} (2x^2 + 10) - (4x + 16) \, dx + \int_{-1}^{3} (4x + 16) - (2x^2 + 10) \, dx + \int_{3}^{5} (2x^2 + 10) - (4x + 16) \, dx
\]

\[
= \frac{14}{3} + \frac{64}{3} + \frac{64}{3} = \frac{142}{3}
\]

9.

**Solution:** Graph it and find the intersection points: \((0, 0)\), \((1, 7)\), and \((2, 4)\).
\[ A = \int_0^1 7x - 2x \, dx + \int_1^2 (8 - x^2) - 2x \, dx = \frac{31}{6} \]

You can also do a \( dy \) integral:

\[ A = \int_0^4 \left( \frac{y}{2} - \frac{y}{7} \right) \, dy + \int_0^4 \left( \sqrt{8 - y} - \frac{y}{7} \right) \, dy \]