Math 132 Exam 1 Review

Solutions

1. Find the most general antiderivative of the function: $f(x) = \dfrac{2+x^2}{1+x^2}$

   Solution: $x + \arctan x + c$

2. Find the most general function f satisfying $f'(x) = \dfrac{2}{1+x^2} - \dfrac{x\cos x + \sqrt{x}}{x}$.

   Solution: $2\arctan(x) - \sin x - 2\sqrt{x} + C$

3. Compute $\int_{-1}^{1}(5|x| + \pi)dx$.

   Solution: $5 + 2\pi$

4. Find the integral associated to the right Riemann sum
   $R_n = \sum_{1}^{n}\left(\left(1+\dfrac{2i}{n}\right)^2 + 1\right)\dfrac{2}{n}$

   Solution: $\int_1^3 (x^2+1)\ dx$

5. Calculate the integral

$$\int_0^2 |x^2 - 1|dx$$

   Solution: 2

6. If $\int_a^c f(x)\ dx = 3$, $\int_a^b f(x)\ dx = -7$, $\int_b^c g(x)\ dx = 6$, find $-\int_c^b 10f(x) + 4g(x)\ dx$.

   Solution: 124

7. Find the definite integral

$$\int_0^{ln2} \frac{e^t}{e^{2t} + 2e^t + 1}dt.$$

   Solution: 1/6

8. Evaluate the definite integral $\int_{-2}^{2} e^{2u+1}\ du$.

   Solution: $\frac{1}{2}(e^5 - e^{-3})$

9. Compute $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\ dx$.

   Solution: 1

10. Compute T_3 for the function $f(x) = 3x^2 + 4x + 6$ for the interval $[0, 3]$. (Recall that $T_3 = (L_3 + R_3)/2$.)

    Solution: 64.5

11. Suppose $\int_1^3 f(x)\ dx = 6$ and $\int_1^3 g(x)\ dx = 2$. What is $\int_1^3 (2f(x) - 3g(x))\ dx$?

    Solution: 6

12. Suppose $\int_1^4 f(x) = 8$, $\int_1^2 f(x) = 4$, $\int_3^4 f(x) = 2$, what is $\int_2^3 f(x)$?

    Solution: 2

13. Find $\int_0^1 x\left(\sqrt{x} + \sqrt[3]{x}\right)\ dx$

    Solution: 29/35

14. If $g(x) = \int_1^{\sqrt{x}} \sin(t^2)\ dt$, find $g'(x)$.

    Solution: $\dfrac{\sin x}{2\sqrt{x}}$

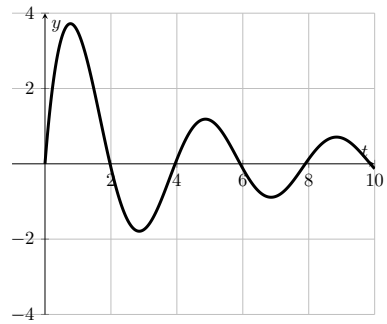
15. Let $g(x) = \int_0^x f(t)\ dt$ where $f(t)$ is the graph below. Determine which of the statements are true:

(a) g attains an absolute maximum at $x = 2$ **Correct**

(b) g has a local maximum at $x = 5$

(c) g has a local minimum at $x = 4$ **Correct**

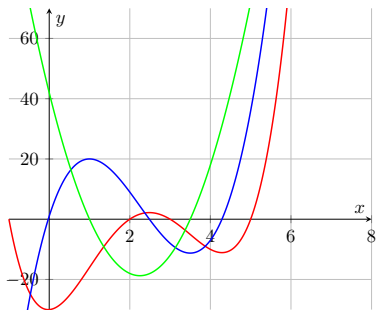
(d) g is concave down on $[0, 2]$



16. Suppose $f''(x) = -9\sin 3x$ and $f'(0) = 0$ and $f(0) = 2$. Find $f(\pi/4)$.
Solution: $-3\pi/4 + 1/\sqrt{2} + 2$

17. If $f'''(x) = \sin x$, $f(0) = -3$, $f'(0) = 4$ and $f''(0) = 1$. What is $f(x)$?
Solution: $\cos x + x^2 + 4x - 4$

18. The three graphs below are f, f' and f''. Identify which is which.



Solution: Red is f, blue is f' and green is f''

19. Write $\int_2^{10} x^6\, dx$ as a limit of Riemann Sums (right handed sums). (Your answer should be in summation notation.)
Solution: $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{8i}{n}\right)^6 \cdot \frac{8}{n}$

20. Suppose you know that $\displaystyle\int_0^b f(x)\, dx = \ln(b+1)$ for $b > 0$. What is $\displaystyle\int_3^5 \big(3f(x) - 2\big)\, dx$?
Solution: $3\ln(3/2) - 4$
Note: this was changed from $\int_2^b f(x)\, dx$ to $\int_0^b f(x)\, dx$. Without this change there is a clear issue with $b = 2$ (which should give an integral of 0, not $\ln 3$.

21. Find a function $F(x)$ such that $F''(x) = 4 + 6x + 24x^2$, $F(0) = 3$, $F(1) = 10$.
Solution: $F(x) = 2x^2 + x^3 + 2x^4 + 2x + 3$
Note: this problem is not a typo, solve it as written.

22. Find $\int_{-10}^{6} |3x - 2|\, dx$
Solution: $640/3$

23. $\displaystyle\int_0^5 \frac{1}{3}x^3\, dx = \lim_{n\to\infty} R_n$, where R_n is the right hand Riemann sum. Find R_n.
Solution: $R_n = \dfrac{625n^2 + 1250n + 625}{12n^2}$
One student proposed that the above is wrong and should instead be: $R_n = \dfrac{625n^4 + 1250n^2 + 625}{12n^4}$ I haven't had time to double check it.

24. $\int_1^2 2x^2+1\,dx = \lim_{n\to\infty} R_n$, where R_n is the right hand Riemann sum. Find R_n.

Solution:

$$R_n = 3 + \frac{2(n+1)}{n} + \frac{(n+1)(2n+1)}{3n^2}$$

25. Let $g(x) = x^3$. Find the Riemann sum L_4 for $g(x)$ on the interval $[1,3]$.

Solution: 14

26. Evaluate the following limit by first recognizing it as a Riemann sum for a function defined on $[0,1]$

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$$

Solution: $\int_0^1 \sqrt{x}\,dx = 2/3$

27. Let $g(x) = \int_x^{x^2} \tan t\,dt$. Find $g'(x)$.

Solution: $g'(x) = 2x\tan(x^2) - \tan x$

28. Let $F(x) = \int_{\tan x}^{\sec x} \sqrt{t^2+3}\,dx$. Evaluate $F'(0)$.

Solution: $F'(x) = \sec x \tan x\sqrt{\sec^2 x + 3} - \sec^2 x\sqrt{\tan^2 x + 3}$.
$F'(x) = -\sqrt{3}$.

29. If $f(x) = \int_0^x (4-t^2)e^{t^3}\,dt$, on what interval(s) is f decreasing and on what intervals is f increasing?

Solution: Decreasing on $(-\infty,-2)$ and $(2,\infty)$, increasing on $(-2,2)$.

30. Find all values of x where $F(x) = \int_0^x \frac{t^3-3t^2+2t}{e^t}\,dt$ has a local maximum or local minimum.

Solution: $F'(x) = \frac{x^3-3x^2+2x}{e^x}$
Solving $F'(x) = 0$ gives $x = 0,1,2$. Testing these points gives $x = 0$ is a min, $x = 1$ is a max, $x = 2$ is a min.

31. Find an antiderivative of $e^{x^2}\sin(3x^2+\ln x)$.

Solution: $\int_0^x e^{x^2}\sin(3x^2+\ln x)\,dx$

32. T/F. If $f(x)$ is continuous and has a minimum of 3 on $[2,4]$ then we can conclude $\int_2^4 f(x)\,dx \geq 6$.

Solution: True

33. T/F. Given $\int_1^4 g(t)\,dt = -5$ and $\int_4^3 g(t)\,dt = 2$ then $\int_3^1 g(t)\,dt = -7$.

Solution: False

34. Find $\int_{\pi/3}^b \sin t\,dt$ for any b.

Solution: $\frac{1}{2} - \cos b$

35. If $f(5) = 10$ and $\int_5^{100} f'(x)\,dx = 73$, what is $f(100)$?

Solution: $f(100) - f(5) = 73$ so $f(100) = 83$

36. Find the value of t where $f(t)$ has a local maximum:
$f(t) = \int_0^t \frac{2x^2+x-10}{1+\sin^2 x}\,dx$.

Solution: Solve $f' = 0$ to get $t = 2$ and $t = -5/2$. These these to find that f has a local max at $t = -5/2$ and a local min at $t = 2$.

37. Find an antiderivative of $\frac{1}{\sqrt{x}} + e^{2x} + \sin 3x$.

Solution: $2\sqrt{x} + \frac{1}{2}e^{2x} - \frac{1}{3}\cos 3x + C$.

38. Suppose you know the following about a function $f(x)$.

x	0	1	2	3	4	5	6	7	8
$f(x)$	-1	3	1	2	2	1	-1	-1	1

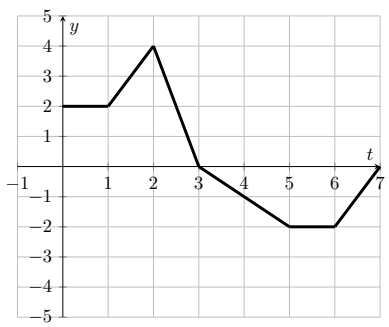
Find the Riemann Sum for $\int_2^6 f(x)\,dx$ using 4 subintervals and right endpoints as sample points.

Solution: $R_4 = 1(f(3) + f(4) + f(5) + f(6)) = 2 + 2 + 1 + (-1) = 4$

39. If x_i^* is a sample point from the ith subinterval of a regular partition of $[1, 3]$ into n subintervals, and Δx is the length of each subinterval, find: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{2}(x_i^*)^2 \Delta x$.

Solution: This is equal to $\int_1^3 \frac{3}{2}x^2\,dx = 13$

40. Let $g(x) = \int_0^x f(t)\,dt$ where f is the function shown. Find $g(5)$.



Solution: 5

41. Let $h(x) = \displaystyle\int_1^{1/x} \sqrt{1 + u^3}\,du$. Find $h'(1/2)$.

Solution: -12

42. Find $\displaystyle\lim_{h\to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt$

Solution:

$$\lim_{h\to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt = \frac{d}{dx}\bigg|_{x=2} \int_2^{x} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt = x^2 \sin\left(\frac{\pi x}{4}\right)\,dt\bigg|_{x=2} = 4\sin\left(\frac{\pi}{2}\right)\,dt = 4$$

43. Calculate $\displaystyle\int_1^2 \frac{e^x}{1 - e^x}dx$

Solution: $\ln(e - 1) - \ln(e^2 - 1)$

44. Let $u = 2t - 1$ and rewrite the integral in the variable u.

$$\int_2^3 t\sqrt{2t - 1}\,dt$$

Solution: $\displaystyle\int_3^5 \frac{1}{4}(u^{3/2} + u^{1/2})\,dy$

45. Find $\displaystyle\int \frac{dx}{x \ln x} = \ln(\ln x) + C$

46. Compute $\displaystyle\int \frac{x}{x^2 - 3}\,dx = \frac{1}{2}\ln(x^2 - 3) + C$

47. Compute $\displaystyle\int_0^{\pi/3} \sin x\,\cos^4 x\,dx = \frac{31}{160}$

48. Compute $\displaystyle\int_0^{\sqrt{\ln 3}} 3x\, e^{-x^2}\, dx = 1$

49. Compute $\displaystyle\int_0^1 \sqrt{x^2 - x^4}\, dx = \frac{1}{3}$

50. Evaluate the definite integral $\displaystyle\int_0^2 \frac{dx}{\sqrt{2x+5}} = 3 - \sqrt{5}$

51. Find $\displaystyle\int \cot(x)\ln(\sin x)\, dx = \frac{1}{2}(\ln(\sin x))^2 + C$

52. Find $\displaystyle\int_0^{\pi/2} e^{\sin x}\cos x\, dx = e - 1$

53. Find $\displaystyle\int_1^2 x\sqrt{x-1}\, dx$

    Solution: $\frac{16}{15}$

54. Find $\displaystyle\int_1^{e^b} \frac{\cos(\ln t)}{t}\, dt$

    Solution: $\sin b$