1. Find the most general antiderivative of the function: \( f(x) = \frac{2 + x^2}{1 + x^2} \)
   Solution: \( x + \arctan x + c \)
2. Find the most general function \( f \) satisfying \( f'(x) = \frac{2}{1 + x^2} - \frac{x \cos x + \sqrt{x}}{x} \).
   Solution: \( 2 \arctan(x) - \sin x - \frac{2}{\sqrt{x}} + C \)
3. Compute \( \int_{-1}^{1} (5|x| + \pi) \, dx \).
   Solution: \( 5 + \frac{2\pi}{3} \)
4. Find the integral associated to the right Riemann sum \( R_n = \sum_{i=1}^{n} \left( \left( 1 + \frac{2i}{n} \right)^2 + 1 \right) \frac{2}{n} \).
   Solution: \( \int_{1}^{3} (x^2 + 1) \, dx \)
5. Calculate the integral \( \int_{0}^{2} |x^2 - 1| \, dx \).
   Solution: \( 2 \)
6. If \( \int_{a}^{c} f(x) \, dx = 3, \int_{a}^{b} f(x) \, dx = -7, \int_{b}^{c} g(x) \, dx = 6, \) find \( -\int_{c}^{b} 10f(x) + 4g(x) \, dx \).
   Solution: \( 124 \)
7. Find the definite integral \( \int_{0}^{\ln 2} \frac{e^t}{e^{2t} + 2e^t + 1} \, dt \).
   Solution: \( \frac{1}{6} \)
8. Evaluate the definite integral \( \int_{-2}^{2} e^{2u+1} \, du \).
   Solution: \( \frac{1}{2} (e^5 - e^{-3}) \)
9. Compute \( \int_{0}^{\pi/3} \frac{\sin x}{\cos^2 x} \, dx \).
   Solution: \( 1 \)
10. Compute \( T_3 \) for the function \( f(x) = 3x^2 + 4x + 6 \) for the interval \([0, 3] \). (Recall that \( T_3 = (L_3 + R_3) / 2 \).)
    Solution: \( 64.5 \)
11. Suppose \( \int_{1}^{3} f(x) \, dx = 6 \) and \( \int_{1}^{3} g(x) \, dx = 2 \). What is \( \int_{1}^{3} (2f(x) - 3g(x)) \, dx \)?
    Solution: \( 6 \)
12. Suppose \( \int_{1}^{3} f(x) = 8, \int_{1}^{2} f(x) = 4, \int_{3}^{4} f(x) = 2, \) what is \( \int_{2}^{3} f(x) \)?
    Solution: \( 2 \)
13. Find \( \int_{0}^{1} x (\sqrt{x} + \sqrt[3]{x}) \, dx \)
    Solution: \( 29/35 \)
14. If \( g(x) = \int_{1}^{x} \sin(t^2) \, dt \), find \( g'(x) \).
    Solution: \( \frac{\sin x}{2\sqrt{x}} \)
15. Let \( g(x) = \int_{0}^{x} f(t) \, dt \) where \( f(t) \) is the graph below. Determine which of the statements are true:
(a) \( g \) attains an absolute maximum at \( x = 2 \) \[\text{Correct}\]

(b) \( g \) has a local maximum at \( x = 5 \)

(c) \( g \) has a local minimum at \( x = 4 \) \[\text{Correct}\]

(d) \( g \) is concave down on \([0, 2]\)

16. Suppose \( f''(x) = -9 \sin 3x \) and \( f'(0) = 0 \) and \( f(0) = 2 \). Find \( f(\pi/4) \).
   \[\text{Solution:} \quad -3\pi/4 + 1/\sqrt{2} + 2\]

17. If \( f'''(x) = \sin x \), \( f(0) = -3 \), \( f'(0) = 4 \) and \( f''(0) = 1 \). What is \( f(x) \)?
   \[\text{Solution:} \quad \cos x + x^2 + 4x - 4\]

18. The three graphs below are \( f \), \( f' \) and \( f'' \). Identify which is which.

   \[\text{Solution:} \quad \text{Red is } f, \text{ blue is } f' \text{ and green is } f''\]

19. Write \( \int_2^{10} x^6 \, dx \) as a limit of Riemann Sums (right handed sums). (Your answer should be in summation notation.)
   \[\text{Solution:} \quad \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{8i}{n} \right)^6 \cdot \frac{8}{n}\]

20. Suppose you know that \( \int_0^b f(x) \, dx = \ln(b + 1) \) for \( b > 0 \). What is \( \int_3^5 (3f(x) - 2) \, dx \)?
   \[\text{Solution:} \quad 3 \ln(3/2) - 4\]
   Note: this was changed from \( \int_2^b f(x) \, dx \) to \( \int_0^b f(x) \, dx \). Without this change there is a clear issue with \( b = 2 \) (which should give an integral of 0, not \( \ln 3 \)).

21. Find a function \( F(x) \) such that \( F''(x) = 4 + 6x + 24x^2 \), \( F(0) = 3 \), \( F(1) = 10 \).
   \[\text{Solution:} \quad F(x) = 2x^2 + x^3 + 2x^4 + 2x + 3\]
   Note: this problem is not a typo, solve it as written.

22. Find \( \int_{-10}^6 |3x - 2| \, dx \)
   \[\text{Solution:} \quad 640/3\]

23. \( \int_0^5 \frac{1}{3} x^3 \, dx = \lim_{n \to \infty} R_n \), where \( R_n \) is the right hand Riemann sum. Find \( R_n \).
   \[\text{Solution:} \quad R_n = \frac{625n^2 + 1250n + 625}{12n^2}\]
   One student proposed that the above is wrong and should instead be: \( R_n = \frac{625n^4 + 1250n^2 + 625}{12n^4} \) I haven’t had time to double check it.
24. \[ \int_1^2 2x^2 + 1 \, dx = \lim_{n \to \infty} R_n, \] where \( R_n \) is the right hand Riemann sum. Find \( R_n \).

**Solution:**
\[ R_n = 3 + \frac{2(n + 1)}{n} + \frac{(n + 1)(2n + 1)}{3n^2} \]

25. Let \( g(x) = x^3 \). Find the Riemann sum \( L_4 \) for \( g(x) \) on the interval \([1, 3]\).

**Solution:** 14

26. Evaluate the following limit by first recognizing it as a Riemann sum for a function defined on \([0, 1]\)
\[ \lim_{n \to \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}} \right) \]

**Solution:** \( \int_0^1 \sqrt{x} \, dx = \frac{2}{3} \)

27. Let \( g(x) = \int_x^2 \tan t \, dt \). Find \( g'(x) \).

**Solution:** \( g'(x) = 2x \tan(x^2) - \tan x \)

28. Let \( F(x) = \int_{\sec x}^{\tan x} \sqrt{t^2 + 3} \, dt \). Evaluate \( F'(0) \).

**Solution:**
\[ F'(x) = \sec x \tan x \sqrt{\sec^2 x + 3} - \sec^2 x \sqrt{\tan^2 x + 3} \]

\[ F'(x) = -\sqrt{3} \]

29. If \( f(x) = \int_0^x (4 - t^2)e^{t^3} \, dt \), on what interval(s) is \( f \) decreasing and on what intervals is \( f \) increasing?

**Solution:** Decreasing on \((-\infty, -2)\) and \((2, \infty)\), increasing on \((-2, 2)\).

30. Find all values of \( x \) where \( F(x) = \int_0^x \frac{t^3 - 3t^2 + 2t}{e^t} \, dt \) has a local maximum or local minimum.

**Solution:**
\[ F'(x) = e^x \left( x^3 - 3x^2 + 2x \right) \]
Solving \( F'(x) = 0 \) gives \( x = 0, 1, 2 \). Testing these points gives \( x = 0 \) is a min, \( x = 1 \) is a max, \( x = 2 \) is a min.

31. Find an antiderivative of \( e^{x^2} \sin(3x^2 + \ln x) \).

**Solution:**
\[ \int_0^x e^{x^2} \sin(3x^2 + \ln x) \, dx \]

32. T/F. If \( f(x) \) is continuous and has a minimum of 3 on \([2, 4]\) then we can conclude \( \int_2^4 f(x) \, dx \geq 6 \).

**Solution:** True

33. T/F. Given \( \int_1^4 g(t) \, dt = -5 \) and \( \int_4^3 g(t) \, dt = 2 \) then \( \int_1^3 g(t) \, dt = -7 \).

**Solution:** False

34. Find \( \int_{\pi/3}^b \sin t \, dt \) for any \( b \).

**Solution:** \( \frac{1}{2} - \cos b \)

35. If \( f(5) = 10 \) and \( \int_5^{100} f'(x) \, dx = 73 \), what is \( f(100) \)?

**Solution:** \( f(100) - f(5) = 73 \) so \( f(100) = 83 \)

36. Find the value of \( t \) where \( f(t) \) has a local maximum:
\[ f(t) = \int_0^t \frac{2x^2 + x - 10}{1 + \sin^2 x} \, dx \]

**Solution:** Solve \( f' = 0 \) to get \( t = 2 \) and \( t = -5/2 \). These these to find that \( f \) has a local max at \( t = -5/2 \) and a local min at \( t = 2 \).
37. Find an antiderivative of $\frac{1}{\sqrt{x}} + e^{2x} + \sin 3x$.

**Solution:** $2\sqrt{x} + \frac{1}{2}e^{2x} - \frac{1}{3}\cos 3x + C$.

38. Suppose you know the following about a function $f(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$-1$</td>
<td>$3$</td>
<td>$1$</td>
<td>$2$</td>
<td>$2$</td>
<td>$1$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Find the Riemann Sum for $\int_2^6 f(x) \, dx$ using 4 subintervals and right endpoints as sample points.

**Solution:** $R_4 = 1(f(3) + f(4) + f(5) + f(6)) = 2 + 2 + 1 + (-1) = 4$

39. If $x_i^*$ is a sample point from the $i$th subinterval of a regular partition of $[1, 3]$ into $n$ subintervals, and $\Delta x$ is the length of each subinterval, find: $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{2}(x_i^*)^2 \Delta x$.

**Solution:** This is equal to $\int_1^3 \frac{3}{2}x^2 \, dx = 13$

40. Let $g(x) = \int_0^5 f(t) \, dt$ where $f$ is the function shown. Find $g(5)$.

**Solution:** 5

41. Let $h(x) = \int_1^{1/x} \sqrt{1 + u^3} \, du$. Find $h'(1/2)$.

**Solution:** $-12$

42. Find $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin \left( \frac{\pi t}{4} \right) \, dt$

**Solution:**

$$\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin \left( \frac{\pi t}{4} \right) \, dt = \frac{d}{dx} \int_2^{x} t^2 \sin \left( \frac{\pi t}{4} \right) \, dt \bigg|_{x=2} = 4 \sin \left( \frac{\pi}{2} \right) \, dt = 4$$

43. Calculate $\int_1^2 \frac{e^x}{1 - e^x} \, dx$

**Solution:** $\ln(e - 1) - \ln(e^2 - 1)$

44. Let $u = 2t - 1$ and rewrite the integral in the variable $u$.

$$\int_2^3 t\sqrt{2t-1} \, dt$$

**Solution:** $\int_3^5 \frac{1}{4}(u^{3/2} + u^{1/2}) \, dy$

45. Find $\int \frac{dx}{x \ln x} = \ln(\ln x) + C$

46. Compute $\int \frac{x}{x^2 - 3} \, dx = \frac{1}{2} \ln(x^2 - 3) + C$

47. Compute $\int_0^{\pi/3} \sin x \cos^4 x \, dx = \frac{31}{160}$
48. Compute \( \int_0^{\sqrt{\ln 3}} 3x e^{-x^2} \, dx = 1 \)

49. Compute \( \int_0^1 \sqrt{x^2 - x^4} \, dx = \frac{1}{3} \)

50. Evaluate the definite integral \( \int_0^2 \frac{dx}{\sqrt{2x + 5}} = 3 - \sqrt{5} \)

51. Find \( \int \cot(x) \ln(\sin x) \, dx = \frac{1}{2} (\ln(\sin x))^2 + C \)

52. Find \( \int_0^{\pi/2} e^{\sin x} \cos x \, dx = e - 1 \)

53. Find \( \int_1^2 x \sqrt{x - 1} \, dx \)
    Solution: \( \frac{16}{15} \)

54. Find \( \int_1^b \frac{\cos(ln t)}{t} \, dt \)
    Solution: \( \sin b \)