1. **Clicker** (Method: multiplication)

\[ \log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \]

\[ \frac{1}{2 + x} = \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \frac{x^4}{32} - \cdots \]

Use multiplication of power series to find a series for \( \frac{1}{x + 2} \cdot \ln(1 + x) \).

(This is definitely a pain, but just distribute the terms out.)

\[ \frac{1}{2 + x} \cdot \ln(1 + x) = \left( x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \right) \left( \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \frac{x^4}{32} - \cdots \right) \]

(a) \( \sum_{n=1}^{\infty} \frac{1}{2n+1} x^n \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} x^n \)

(c) \( \sum_{n=1}^{\infty} \frac{1}{2n+1} x^n \)

(d) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{2n+1} x^n \)

(e) Something else

2. (Method: Long Division–Challenging and worth avoiding whenever possible)

Use long division of power series to find a series for \( \frac{\ln(1+x)}{\ln(1-x)} \)

Note: \( \ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \)

and: \( \ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots \)

3. (Method: Integration. Note \( \int \frac{1}{1 + x^2} \, dx = \arctan x + C \))

Find a series for arctan \( x \).

4. Use your previous series to find a series for \( \arctan(1) = \pi/4 \)

5. Find a series for \( \frac{1}{1 + x} \pi \) and use your series to approximate \( \int_0^{0.25} \frac{1}{1 + x^{10}} \, dx \)
Section 11.10: Taylor Series

- Goal: Given a function: find a power series that equal the function
- Taylor Series
  \[ f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n \text{ where } c_n = \frac{f^{(n)}(a)}{n!}. \]
- Maclaurin Series (Taylor Series with \(a = 0\))
  \[ f(x) = \sum_{n=0}^{\infty} c_n x^n \text{ where } c_n = \frac{f^{(n)}(0)}{n!}. \]

6. Let \( f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \)
   - (a) Find \( f(0) \)
   - (b) Find \( f(1) \)
   - (c) Find \( f'(x) = \)
   - (d) Find \( f''(x) = \)
   - (e) Graph \( f(x) \).

7. Find the Taylor Series for \( f(x) = \sin x \) centered at \( x = 0 \)
   (Make a table for the derivatives!)

8. Find the Taylor Series for \( f(x) = \cos x \) centered at \( x = 0 \)

9. Find the Taylor Series for \( f(x) = 4 + 2x - 3x^2 - x^3 + 7x^4 - x^5 \) centered at \( x = 0 \).

10. Find the Taylor Series for \( f(x) = 4 + 2x - 3x^2 - x^3 + 7x^4 - x^5 \) centered at \( x = 1 \).