Section 11.7: Series Strategy

Section 11.8: Power Series

- Goal: Apply what we’ve learned to series like these:
  \[ \sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad \sum_{n=0}^{\infty} a_n (x-a)^n \]

- For what values of \( x \) does a power series converge?
  - Radius of Convergence
  - Interval of Convergence

- For \( \sum a_n (x-a)^n \), three possibilities.
  (i) The series converges for only \( x = a \).
  (ii) The series converges for all \( x \).
  (iii) There is a positive number \( R \) and series converges for \( |x-a| < R \) but diverges for \( |x-a| > R \).

1. [Clicker] Select the easiest \( x \) value to plug in to the power series \( \sum_{n=0}^{\infty} x^n \)
   (a) \( x = 0 \)  (b) \( x = 1 \)  (c) \( x = -1 \)  (d) No value is easy to plug in.

2. [Clicker] Select the easiest \( x \) value to plug in to the power series \( \sum_{n=0}^{\infty} \frac{(x+1)^n}{n^2 + 1} \)
   (a) \( x = 0 \)  (b) \( x = 1 \)  (c) \( x = -1 \)  (d) No value is easy to plug in.

3. Determine the Radius of Convergence and Interval of Convergence for the power series below.
   \[ \sum_{n=0}^{\infty} x^n \]

4. What is the difference between a sequence and a series?

5. What does it mean for a sequence to diverge? Converge?

6. What does it mean for a series to diverge? Converge?

7. What is a partial sum and what are they good for?

8. Give an example of a convergent geometric series.

9. Give an example of a divergent geometric series.

10. What is the Test for Divergence and what is it good for?

11. What is the Integral Test and what is it good for?

12. What is a \( p \) Series?

13. What is the Comparison Test?

14. What is the Limit Comparison Test?

15. What is an Alternating Series?

16. What is the Alternating Series Test?

17. What is Conditionally Convergent?

18. What is the Absolutely Convergent?

19. What is the Ratio Test?

20. What is the Root Test?

21. [Clicker] Which converge:
   I. \( \sum_{n=2}^{\infty} \frac{n-1}{3n+1} \)  II. \( \sum_{n=1}^{\infty} ne^{-n^2} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

22. [Clicker] Which converge?
   I. \( \sum_{n=1}^{\infty} \left(2^n - 1\right)^n \)  II. \( \sum_{n=3}^{\infty} \frac{e^n}{n^2 - 3} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II
23. Clicker Which converge?
   I. \( \sum_{n=2}^{\infty} \frac{1}{\ln n} \)  
   II. \( \sum_{n=1}^{\infty} ne^{-n} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

24. Clicker Which converge conditionally, but not absolutely?
   I. \( \sum_{n=2}^{\infty} \frac{(-1)^nn!}{n^n} \)  
   II. \( \sum_{n=2}^{\infty} \frac{(-1)^n}{n(n\ln n)^3} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

25. Clicker Which converge?
   I. \( a_n = \frac{n^3 + \sqrt{3n^6 + 2n - 1}}{4n^3 + 2n + 5} \)  
   II. \( b_n = \frac{\cos \left( \frac{1}{n} \right)}{n + 1} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

26. Clicker For which of the following series is the Ratio Test inconclusive?
   I. \( \sum_{n=2}^{\infty} \frac{n}{2^n} \)  
   II. \( \sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

27. Clicker Which converge?
   I. \( \sum_{n=2}^{\infty} \left( \frac{n}{n+1} \right)^n \)  
   II. \( \sum_{n=1}^{\infty} \frac{n^{3n}}{n!} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

28. Clicker Which converge?
   I. \( \sum_{n=2}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n} \)  
   II. \( \sum_{n=1}^{\infty} \frac{e^n + n}{e^{3n} - 3} \)
   (a) Neither of them. (b) I only (c) II only (d) I and II

29. Name the strategy(s) to use to determine convergence or divergence:
   (a) \( \sum (0.98)^n \)  
   (b) \( \sum (-5)^{-n} \)  
   (c) \( \sum \frac{1}{n(n+6)} \)  
   (d) \( \sum \frac{(-1)^n}{37n} \)  
   (e) \( \sum ne^{-n^2} \)  
   (f) \( \sum \frac{2^n}{n!} \)  
   (g) \( \sum \frac{n!}{en} \)  
   (h) \( \sum \frac{n^n}{3^{3n+1}} \)