8.1 Arc Length

\[ L = \int ds = \int_a^b \sqrt{1 + (f'(x))^2} \, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \]

8.2 Surface Area

\[ S = \int ds = \int_a^b \frac{2\pi r}{dS} = \int_a^b 2\pi r \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \]

Warm-up Problems

1. Clicker

   Below is a graph of \( f(x) = \frac{x}{\pi + x^2} \).

   
   ![Graph of f(x) = x/(\pi + x^2)]

   Find \( \int_{-\infty}^{\infty} \frac{x}{9 + x^2} \, dx \)

   (a) Converges to 0  
   (b) Converges to something else  
   (c) Diverges to +\( \infty \)  
   (d) Diverges to -\( \infty \)  
   (e) Diverges

   Note the following work:

   \[
   \int_0^\infty \frac{x}{9 + x^2} \, dx = \lim_{t \to \infty} \int_0^t \frac{x}{9 + x^2} \, dx = \lim_{t \to \infty} \left[ \frac{1}{2} \ln(x^2 + 9) \right]_0^t \\
   = \lim_{t \to \infty} \left[ \frac{1}{2} \ln(t^2 + 9) - \frac{1}{2} \ln(9) \right] = \lim_{t \to \infty} \frac{1}{2} \left[ \ln \left( \frac{t^2 + 9}{9} \right) \right] = \infty
   \]

2. Find the length of the line segment between the points \( (x_i, f(x_i)) \) and \( (x_j, f(x_j)) \).

3. Clicker

   Find the surface area of the cone

   (a) \( \pi rl \)  
   (b) \( 2\pi rl \)  
   (c) \( \pi(r^2 + l^2) \)  
   (d) \( \pi \sqrt{r^2 + l^2} \)
4. **Clicker**

Find the surface area of the frustum of the cone. (Just the cone part, not the top/bottom.)

(a) $\pi l r_1$
(b) $\pi l r_2$
(c) $2\pi l \left( \frac{r_1 + r_2}{2} \right)$
(d) $2\pi l \left( \frac{r_1 - r_2}{2} \right)$

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**Class Problems**

5. Set up arc length integrals

(a) Find length of $y = 2x + 3$ between $x = 0$ and $x = 2$
(b) Find length of $y = \cos x$ between $x = 0$ and $x = \pi$
(c) Find length of $y = 2(x - 1)^{3/2}$ from $x = 1$ to $x = 5$.

6. Set up surface area integrals

(a) Find the area of $y = \sqrt{x}$ on the interval $[3/4, 15/4]$ rotated about the $x$-axis.
(b) Find the area of $x = \frac{y^3}{3}$ on the interval $y = 0$ to $y = 1$ rotated about the $y$-axis.
(c) Find the area of $y = \sin x$ from $x = 0$ to $x = \pi$. 