7.3 Trig Substitution
• Completing the square

7.4 Partial Fractions
• Polynomial Long Division

Warm-up Problems

1. What is a rational function?

2. What are the trig identities used in trig substitution?

3. **Clicker** Complete the square $x^2 - 6x + 13 = ???$
   (a) $x^2 - 6x + 13 = (x + 3)^2 + 4$
   (b) $x^2 - 6x + 13 = (x - 3)^2 + 4$
   (c) $x^2 - 6x + 13 = (x - 3)^2 + 4$
   (d) $x^2 - 6x + 13 = (x + 3)^2 + 4$

Class Problems

4. \[ \int_3^5 \frac{1}{\sqrt{x^2 - 6x + 13}} \, dx \]
   Hint: Use the completed square from Problem 3.

5. \[ \int \sqrt{x^2 + 4x + 5} \, dx \]

6. \[ \int_0^1 \sqrt{4 - x^2} \, dx \]

7. \[ \int_1^2 \frac{1}{x^2\sqrt{x^2 + 16}} \, dx \]

8. \[ \int_0^4 x^2\sqrt{16 - x^2} \, dx \]

9. Perform polynomial long division and write your answers correctly.
   (a) \( \frac{x^2 + 1}{x + 2} \)
   (b) \( \frac{x^2 + 1}{x^2 + 2} \)
   (c) \( \frac{2x^3 - x^2 + 5x + 1}{3x + 2} \)
   (d) \( \frac{2x^5 - x^2 + 5x + 1}{(x - 2)(x^2 + 1)} \)
10. Find the general integrals below. (These are the types of integrals that should appear when partial fractions is conducted correctly.)

(a) \[ \int \frac{1}{ax + b} \, dx \]

(b) \[ \int \frac{1}{(ax + b)^2} \, dx \]

(c) \[ \int \frac{x}{x^2 + a^2} \, dx \]

(d) \[ \int \frac{x}{(x^2 + a^2)^2} \, dx \]

(e) \[ \int \frac{1}{x^2 + a^2} \, dx = \]

(f) \[ \int \frac{1}{(x^2 + a^2)^2} \, dx \]