1. Calculate \( \int_1^2 \frac{e^x}{1 - e^x} \, dx \)

\[ \text{Solution: } \ln(e - 1) - \ln(e^2 - 1) \]

2. Let \( u = 2t - 1 \) and rewrite the integral in the variable \( u \).

\[ \int_2^3 t\sqrt{2t - 1} \, dt \]

\[ \text{Solution: } \int_5^3 \frac{1}{4}(u^{3/2} + u^{1/2}) \, dy \]

3. Find \( \int \frac{dx}{x \ln x} = \ln(\ln x) + C \)

4. Compute \( \int \frac{x}{x^2 - 3} \, dx = \frac{1}{2} \ln(x^2 - 3) + C \)

5. Compute \( \int_{\pi/3}^{\ln 3} \sin x \cos^4 x \, dx = \frac{31}{160} \)

6. Compute \( \int_0^3 3x e^{-x^2} \, dx = 1 \)

7. Compute \( \int_0^1 \sqrt{x^2 - x^4} \, dx = \frac{1}{3} \)

8. Evaluate the definite integral \( \int_0^2 \frac{dx}{\sqrt{2x + 5}} = 3 - \sqrt{5} \)

9. Find \( \int \cot(x) \ln(\sin x) \, dx = \frac{1}{2}(\ln(\sin x))^2 + C \)

10. Compute the area under \( y = \sqrt{x + 1} \) from \( x = 0 \) to \( x = 3 \).

\[ \text{Solution: } \frac{14}{3} \]

11. Let \( R \) be the region bounded by \( x = 1, x = 2 \) and \( y = 3x - 1 \).

Find the volume of the solid obtained by rotating the region \( R \) about the \( x \)-axis.

\[ \text{Solution: } 13\pi \]

12. Find \( \int_0^{\pi/2} e^{\sin x} \cos x \, dx = e - 1 \)

13. Find \( \int_1^2 x\sqrt{x-1} \, dx \)

\[ \text{Solution: } \frac{16}{15} \]

14. Find the area enclosed by \( y = x^2 \) and \( y = x \).

\[ \text{Solution: } \frac{1}{6} \]

15. Find the area enclosed by \( y^2 = x + 6 \) and \( y = x \).

\[ \text{Solution: } 20 + \frac{5}{6} \]

16. Find the area enclosed by \( y = \frac{\ln x}{x} \) and \( y = \frac{(\ln x)^2}{x} \).

\[ \text{Solution: } \frac{1}{6} \]

17. Find the volume of the solid whose base is the disc centered at the origin with radius one, whose cross sections perpendicular to the \( x \)-axis are squares.

\[ \text{Solution: } \frac{16}{3} \]
18. Find the volume of the solid whose base is the region $|x| + |y| \leq 1$ and whose vertical cross sections perpendicular to the $y$ axis are semicircles (with diameter along the base).

Solution: $\frac{\pi}{3}$

19. Find the volume of the solid obtained by rotating about the $y$-axis the region bounded by $y = x^3$, $y = 8$ and $x = 0$.

Solution: $96\pi/5$

20. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the $x$-axis.

Solution: $3\pi/10$

21. Find the volume of the solid obtained by rotating the region bounded by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$ about the horizontal line $y = 3$.

Solution: $\pi(6\sqrt{2} - \frac{13}{2})$

22. Find the volume of the solid obtained by rotating the region bounded by $y = 4 - x^2$, $x = 0$, and $x = 1$ about the vertical line $x = 2$.

Solution: $3\pi/10 + \frac{13\pi}{6}$

23. $\int_0^3 \frac{1}{\sqrt{x}} \, dx = 2\sqrt{3}$

24. Find the average value of $|x^2 - 2|$ on $[0, 2]$.

Solution: $(2/3)(2\sqrt{2} - 1)$

25. Find $\int_1^c \frac{\cos(ln t)}{t} \, dt = \sin b$

26. $\int_0^1 te^{\pi t} \, dt = (\pi e^\pi - e^\pi + 1)/(\pi^2)$

27. Find the average value of $y = x^2$ over $[1, 3]$.

Solution: $\frac{13}{3}$

28. Find the number $c$ for which $\sqrt{c}$ is the average value of $\sqrt{x}$ over the interval $[0, 2]$.

Solution: $C = 8/9$

29. Find the average value of $e^x$ on $[0, \ln 2]$ Solution: $1/\ln 2$

30. Find $\int_0^1 \log_2 x \, dx = 2 - 1/\ln 2$

31. $\int_0^{\pi/2} x \sin x \, dx = 1$

32. $\int_0^1 x^2 e^{-x} \, dx = 2 - \frac{5}{e}$

33. $16 \int_1^e x^3 \ln x \, dx = 3e^4 + 1$

34. $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx = 2/15$

35. Let $R$ be region above $x$-axis and below $y = (\sin x)/x$, $0 \leq x \leq \pi/2$. Rotate $R$ about $y$ axis and find volume.

Solution: $2\pi$

36. Suppose $f(x) = x^2$ and that $f(7)$ is equal to the average value of $f$ on the interval $[2, b]$. What is $b$?

Solution: 11

37. $\int_0^{\pi/2} t \cos t \, dt = (\pi - 2)/2$

38. $\int_0^{\pi/4} \sqrt{\sec^2 x - 1} \, dx = \ln \sqrt{2}$

39. $\int \arcsin x \, dx = x \arcsin x + \sqrt{1 - x^2} + C$

Note: $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

40. Let $R$ be the region in the first quadrant enclosed by $y = x^2 + 2$, $y = 6$ and $x = 0$. Rotate $R$ about the $x$ axis. Using shell method, find the integral representing the volume.

Solution: $2\pi \int_0^2 y\sqrt{y-2} \, dy$

41. Suppose we know $f(1) = 0$, $f(2) = 1$, $\int_1^2 f(x) \, dx = -2$. Use integration by parts to find $\int_1^2 x f'(x) \, dx$ Solution: 4
42. \( \int_0^{\pi/4} \tan^3 x \; dx = (1 - \ln 2)/2 \)

43. Find \( \int_0^{\pi/4} \tan^2 x \sec^4 x \; dx = 8/15 \)

44. Find \( \int_0^\pi \sin^2 x \; dx = \pi/2 \)

45. Find \( \int_0^\pi \cos^2 x \; dx = \pi/2 \)

46. Find \( \int_0^\pi \sin^4 x \cos^2 x \; dx = \pi/16 \)

47. Find \( \int_0^{\pi/3} \sec^2 \theta \; d\theta = \sqrt{3} \)

48. Let \( f(x) = x^2 + 1 \). Find the point \( c \) in \([1, 7]\) such that \( f(c) \) is the average value of \( f \) on \([1, 7]\).

**Solution:** \( c = \sqrt{19} \)

49. Find \( \int_1^2 \log_a(x) \; dx \)

**Solution:** Use change of base log \( a \) \( x = \frac{\ln x}{\ln a} \).

50. Find \( \int_0^{\pi/2} x \sin x \; dx = 1 \)

51. Find \( \int \sin^3 x \; dx = \frac{1}{3} \cos^3 x - \cos x + C \).