Trig Identities:

- $1 + \tan^2 \theta = \sec^2 \theta$
- $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$
- $\sin A \cos B = \frac{1}{2} (\sin(A + B) + \sin(A - B))$
- $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2} (1 + \cos 2x)$

Trig Integral Strategies

<table>
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<tr>
<th>Integral</th>
<th>Strategy</th>
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| $\int \sin^m x \cos^p x \, dx$ | - If $m, n$ (or both) are odd. Use either $u = \sin x$ or $u = \cos x$.
  - If $m, n$ are both even, use half angle identities.
| $\int \tan^m x \sec^n x \, dx$ | - Use either $u = \tan x$ or $u = \sec x$
  - If the above doesn’t work, try something else.
| $\int \sin(mx) \cos(nx) \, dx$ | - Use product to sum formulas
| $\int \sec x \, dx$ or $\int \csc x \, dx$ | - This is a trick. Just be ready to look this integral up if needed.

Warm-up Problems

1. Integrate with $u = \sin x$: $\int \sin^4 x \cos^5 x \, dx$
2. Integrate with $u = \cos x$: $\int \sin^3 x \cos^5 x \, dx$
3. Integrate with $u = \tan x$: $\int \tan x \sec^5 x \, dx$
4. Integrate with $u = \sec x$: $\int \tan x \sec^6 x \, dx$
5. Integrate using the product to sum formulas above: $\int \sin(2x) \sin(17x) \, dx$

Class Problems

6. **Clicker** Identify the strategy to integrate the integrals below

   **Answers for all parts of this question**

   A. Let $u = \sin x$.
   B. Let $u = \cos x$
   C. Let $u = \tan x$
   D. Let $u = \sec x$
   E. Some other trig identity, something tricky or maybe its impossible.

   (a) $\int \sin^4 x \cos^5 x \, dx$
   (b) $\int \cos^2 x \, dx$
   (c) $\int \sec^4 x \, dx$
   (d) $\int \tan^4 x \, dx$
   (e) $\int \sin^5 x \cos^4 x \, dx$
   (f) $\int \sin^4 x \cos^6 x \, dx$

7. Compute the integrals in Problem 6
8. Below are a variety of trig integral problems. Some are routine, some are tedious, and some (I think) are more difficult.

(a) \[ \int \sin^3 x \cos^2 x \, dx \]

(b) \[ \int \sin^2 x \cos^7 x \, dx \]

(c) \[ \int \sin^4 x \, dx \]

(d) \[ \int \sin^6 x \cos^2 x \, dx \]

(e) \[ \int \sec^3 x \, dx \]

(f) \[ \int \sec^4 x \, dx \]

(g) \[ \int \tan^2 x \, dx \]

(h) \[ \int \tan^4 x \, dx \]

(i) \[ \int \frac{\cos^2 x}{1 + \sin x} \, dx \]

(j) \[ \int (\sec^2 x) \sqrt{5 + \tan x} \, dx \]

(k) \[ \int \frac{\cos^5 x \sin x}{1 - \sin^2 x} \, dx \]

(l) \[ \int \frac{\sin 2x}{\cos^4 x + \sin^4 x} \, dx \]