Math 132 - September 27, 2017

6.5: Average Value and MVT for Integrals **7.1: Integration by Parts**

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Average Value

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

MVT for integrals. If f is continuous on $[a,b]$ then there is a number $c \in [a,b]$ such that:

$$f(c) = f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Pick all that are differentiation rules.
   (a) Chain Rule
   (b) Product Rule
   (c) Substitution Rule
   (d) Quotient Rule
   (e) Parts Rule
2. Write down all other differentiation rules you can think of.
3. Let $f(x) = x^2$.

i	1	2	3	4	5
$\mathbf{x_i}$	1	2	3	4	5
$\mathbf{f(x_i)}$	1^2	2^2	3^2	4^2	5^2

   Find the average of the values $f(x_i)$ in the chart.
4. Almost the same as previous problem

i	1	2	3	4	5	6	7	8	9	10
$\mathbf{x_i}$	.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$\mathbf{f(x_i)}$	$.5^2$	1^2	1.5^2	2^2	2.5^2	3^2	3.5^2	4^2	4.5^2	5^2

   Find the average of the values $f(x_i)$ in the chart.
5. (Review Problem) Set up integrals using shells and washers for rotating region bounded by $x = (y-2)^2$ and $y = x$ about the line $y = -1$.

Class Problems

6. Find the average value
   (a) x^2 on $[-1, 1]$
   (b) e^x on $[1, 10]$
   (c) $\sin x$ on $[0, \pi]$
   (d) $\cos x$ on $[0, \pi]$
   (e) $\cos x$ on $[0, \pi/2]$
7. Find the x-value guaranteed by the MVT for integrals in Problem 6.

8. $\boxed{\textbf{Clicker}}$ If u and v and functions and "d" means derivative, the product rule can be written as

$$d(uv) = v\ du + u\ dv$$

Write this in integral form.

(a) $uv = vu + uv$
(b) $\int uv = \int v\ du + \int u\ dv$
(c) $uv = \int v\ du + \int u\ dv$
(d) $\int d(uv) = \int (v\ du - u\ dv)$
(e) $uv = \int v\ du - \int u\ dv$

9. Use integration by parts to compute the integral. Use the given u and dv.

(a) $\displaystyle\int \ln x\ dx$
$u = \ln x,\ dv = dx$

(b) $\displaystyle\int x \cos x\ dx$
$u = x,\ dv = \cos x\ dx$

(c) $\displaystyle\int x \cos x\ dx$
$u = \cos x,\ dv = x\ dx$

10. Use integration by parts to find the indefinite integral

(a) $\int x^2 e^{3x}\ dx$
(b) $\int x^2 \sin(10x)\ dx$
(c) $\int e^x \cos x\ dx$
(d) $\int (\ln x)^2\ dx$
(e) $\int x^3 \cos(x^2)\ dx$