6.3: Volumes with Cylindrical Shells

Shells: \( V = \int_a^b 2\pi rh \, dr \)

Warm-up Problems

1. **Clicker** Find the volume of the “cylindrical shell”.

   - (a) \( 2\pi rh (R - r) \)
   - (b) \( \pi (R^2 - r^2)h \)
   - (c) \( \pi (R + r)h (R - r) \)
   - (d) \( 2\pi \left( \frac{R + r}{2} \right) h \Delta r \)
   - (e) \( \pi r^2 h \)

2. Let \( R \) be the region in the plane bounded by \( y = x^2 \) and \( y = \frac{1}{2} x^2 + 2 \), see below: Rotate \( R \) about the line \( x = 3 \) and find the volume using **washers**.

Class Problems

3. Graph of region bounded by \( y = 4 - x^2 \) and \( y = 0 \) below:

   Rotate this about the line \( x = 3 \) and find the volume using shells.

4. Same region as Problem 3. Find volume if rotated about \( x = -3 \).

5. Same region as Problem 3. Find volume if rotated about \( x = 99 \).
6. Graph of region bounded by $x = y^2$ and $x = 3y^2/4 + 1$ is below: Find the volume of this region rotated about $y = 3$.

7. Find the volume of the region in Problem 6 about the line $x = 4$.

8. [Clicker] Take a region in the plane, $R$, and rotate about the line $y = 18$. The volume integral for washers should look like:

(a) $\int_{x=a}^{x=b} \pi (r_o^2 - r_i^2) \, dx$

(b) $\int_{x=a}^{x=b} \pi (r_o^2 - r_i^2) \, dy$

9. [Clicker] Take a region in the plane, $R$, and rotate about the line $x = 18$. The volume integral for washers should look like:

(a) $\int_{x=a}^{x=b} \pi (r_o^2 - r_i^2) \, dx$

(b) $\int_{x=a}^{x=b} \pi (r_o^2 - r_i^2) \, dy$

10. [Clicker] Take a region in the plane, $R$, and rotate about the line $x = 18$. The volume integral for shells should look like:

(a) $\int_{x=a}^{x=b} 2\pi rh \, dx$

(b) $\int_{x=a}^{x=b} 2\pi rh \, dy$

11. [Clicker] Take a region in the plane, $R$, and rotate about the line $y = 18$. The volume integral for shells should look like:

(a) $\int_{x=a}^{x=b} 2\pi rh \, dx$

(b) $\int_{x=a}^{x=b} 2\pi rh \, dy$

12. Find the volumes.
    Region: bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$, rotated about the $y$-axis.

13. Set up integrals using shells and washers for rotating region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ about the $y$-axis.

14. Set up integrals using shells and washers for rotating region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ about the axis $x = -1$. 