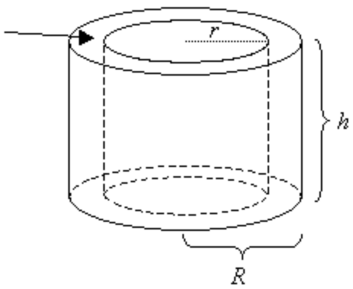
Math 132 - September 25, 2017

6.3: Volumes with Cylindrical Shells

Shells: $V = \int_a^b 2\pi r h \, dr$

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Find the volume of the "cylindrical shell".



(a) $2\pi rh(R-r)$

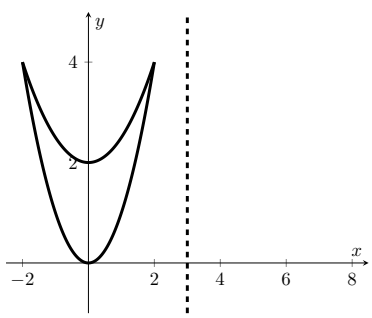
(b) $\pi(R^2-r^2)h$

(c) $\pi(R+r)h(R-r)$

(d) $2\pi\left(\frac{R+r}{2}\right)h\Delta r$

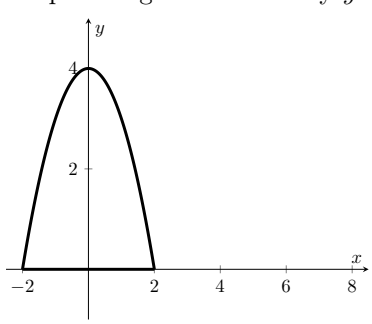
(e) $\pi r^2 h$

2. Let R be the region in the plane bounded by $y = x^2$ and $y = \frac{1}{2}x^2 + 2$, see below: Rotate R about the line $x = 3$ and find the volume using **washers**.



Class Problems

3. Graph of region bounded by $y = 4 - x^2$ and $y = 0$ below:

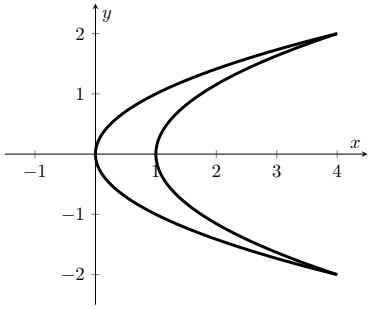


Rotate this about the line $x = 3$ and find the volume using shells.

4. Same region as Probelm 3. Find volume if rotated about $x = -3$.

5. Same region as Probelm 3. Find volume if rotated about $x = 99$.

6. Graph of region bounded by $x = y^2$ and $x = 3y^2/4 + 1$ is below: Find the volume of this region rotated about $y = 3$.



7. Find the volume of the region in Problem 6 about the line $x = 4$.

8. **Clicker** Take a region in the plane, R, and rotate about the line $y = 18$. The volume integral for washers should look like:

   (a) $\displaystyle\int_{x=a}^{x=b} \pi(r_o^2 - r_i^2)\, dx$

   (b) $\displaystyle\int_{x=a}^{x=b} \pi(r_o^2 - r_i^2)\, dy$

   (c) $\displaystyle\int_{y=a}^{y=b} \pi(r_o^2 - r_i^2)\, dx$

   (d) $\displaystyle\int_{y=a}^{y=b} \pi(r_o^2 - r_i^2)\, dy$

   (e) None of these

9. **Clicker** Take a region in the plane, R, and rotate about the line $x = 18$. The volume integral for washers should look like:

   (a) $\displaystyle\int_{x=a}^{x=b} \pi(r_o^2 - r_i^2)\, dx$

   (b) $\displaystyle\int_{x=a}^{x=b} \pi(r_o^2 - r_i^2)\, dy$

   (c) $\displaystyle\int_{y=a}^{y=b} \pi(r_o^2 - r_i^2)\, dx$

   (d) $\displaystyle\int_{y=a}^{y=b} \pi(r_o^2 - r_i^2)\, dy$

   (e) None of these

10. **Clicker** Take a region in the plane, R, and rotate about the line $x = 18$. The volume integral for shells should look like:

    (a) $\displaystyle\int_{x=a}^{x=b} 2\pi rh\, dx$

    (b) $\displaystyle\int_{x=a}^{x=b} 2\pi rh\, dy$

    (c) $\displaystyle\int_{y=a}^{y=b} 2\pi rh\, dx$

    (d) $\displaystyle\int_{y=a}^{y=b} 2\pi rh\, dy$

    (e) None of these

11. **Clicker** Take a region in the plane, R, and rotate about the line $y = 18$. The volume integral for shells should look like:

    (a) $\displaystyle\int_{x=a}^{x=b} 2\pi rh\, dx$

    (b) $\displaystyle\int_{x=a}^{x=b} 2\pi rh\, dy$

    (c) $\displaystyle\int_{y=a}^{y=b} 2\pi rh\, dx$

    (d) $\displaystyle\int_{y=a}^{y=b} 2\pi rh\, dy$

    (e) None of these

12. Find the volumes.
    Region: bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$, rotated about the y-axis.

13. Set up integrals using shells and washers for rotating region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ about the y-axis.

14. Set up integrals using shells and washers for rotating region bounded by $y = \cos x$, $y = 0$, $x = 0$ and $x = \pi/2$ about the axis $x = -1$.