Chapter 6: Applications
Integration = Adding things up
Areas and Volumes

6.1: Areas
To find area between curves:
- Graph region, find intersections
- Decide if you are going to integrate $dx$ or $dy$
- Area $= \int_a^b \text{(Big - Small)} \, dx$

Warm-up Problems

1. Determine where the curves intersect.
   (a) $y = x^2 + x$ and $y = 4x^2 - 20x$
   (b) $x = y^2 - 4$ and $x = 2y - y^2$

Class Problems

2. Find the area between the curves in Problem 1.
3. Given the graphs of the functions below, find integral(s) that represent the area between $y^2 = 4x$ and $y = 2x - 4$.

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(a) $\int_{-2}^{4} \left( \frac{y + 4}{2} - \frac{y^2}{4} \right) \, dy$
(b) $\int_{0}^{1} 2\sqrt{4x} \, dx + \int_{1}^{4} \left( \sqrt{4x} - (2x - 4) \right) \, dx$
(c) $\int_{0}^{4} \sqrt{4x} - (2x - 4) \, dx$
(d) $\int_{-2}^{4} (4y^2 - 2x + 4) \, dy$
(e) $\int_{0}^{4} \left( \frac{y^2}{4} - (2x - 4) \right) \, dx$
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4. **Clicker** Find the area between \( y = \sin x \) and \( y = \cos x \) between \( x = 0 \) and \( x = 2\pi \).

(a) 0 
(b) 2 
(c) \(4\sqrt{2}\) 
(d) 4 
(e) \(-\infty\)

5. Find the area between \( y = x^2 \) and \( y = \sqrt{x} \). 
(Do this in two ways, as a \(dx\) integral and as a \(dy\) integral.)

6. Find the area between \( y = xe^{-x^2} \), \( y = x + 1 \), \( x = 2 \) and the \( y \) axis.

7. Find the area between \( y = 2x^2 + 10 \), \( y = 4x + 16 \), \( x = -2 \) and \( x = 5 \).

8. Find the area enclosed by the graphs of \( y = 8 - x^2 \), \( y = 7x \) and \( y = 2x \) in the first quadrant.