1. Suppose \( \int_{1}^{3} f(x) \, dx = 6 \) and \( \int_{1}^{3} g(x) \, dx = 2 \). What is \( \int_{1}^{3} (2f(x) - 3g(x)) \, dx \)?
   **Solution:** 6

2. Suppose \( \int_{1}^{3} f(x) \, dx = 8 \), \( \int_{1}^{3} f(x) \, dx = 4 \), \( \int_{1}^{3} f(x) \, dx = 2 \), what is \( \int_{2}^{3} f(x) \)?
   **Solution:** 2

3. Find \( \int_{0}^{1} x (\sqrt{x} + \frac{3}{\sqrt{x}}) \, dx \)
   **Solution:** 29/35

4. If \( g(x) = \int_{1}^{\sqrt{x}} \sin(t^2) \, dt \), find \( g'(x) \).
   **Solution:** \( \frac{\sin x}{2\sqrt{x}} \)

5. Let \( g(x) = \int_{0}^{x} f(t) \, dt \) where \( f(t) \) is the graph below. Determine which of the statements are true:
   - (a) \( g \) attains an absolute maximum at \( x = 2 \) **Correct**
   - (b) \( g \) has a local maximum at \( x = 5 \)
   - (c) \( g \) has a local minimum at \( x = 4 \) **Correct**
   - (d) \( g \) is concave down on \([0, 2]\)

6. Suppose \( f''(x) = -9 \sin 3x \) and \( f'(0) = 0 \) and \( f(0) = 2 \). Find \( f(\pi/4) \).
   **Solution:** \(-3\pi/4 + 1/\sqrt{2} + 2\)

7. If \( f'''(x) = \sin x \), \( f(0) = -3 \), \( f'(0) = 4 \) and \( f''(0) = 1 \). What is \( f(x) \)?
   **Solution:** \( \cos x + x^2 + 4x - 4 \)

8. The three graphs below are \( f \), \( f' \) and \( f'' \). Identify which is which.
   **Solution:** Red is \( f \), blue is \( f' \) and green is \( f'' \)

9. Write \( \int_{2}^{10} x^6 \, dx \) as a limit of Riemann Sums (right handed sums). (Your answer should be in summation notation.)
   **Solution:** \( \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{8i}{n} \right)^6 \cdot \frac{8}{n} \)

10. Suppose you know that \( \int_{0}^{b} f(x) \, dx = \ln(b + 1) \) for \( b > 0 \). What is \( \int_{3}^{5} (3f(x) - 2) \, dx \)?
    **Solution:** 3 \( \ln(3/2) - 4 \)

    Note: this was changed from \( \int_{2}^{b} f(x) \, dx \) to \( \int_{0}^{b} f(x) \, dx \). Without this change there is a clear issue with \( b = 2 \) (which should give an integral of 0, not \( \ln 3 \).
11. Find a function \( F(x) \) such that \( F''(x) = 4 + 6x + 24x^2 \), \( F(0) = 3 \), \( F(1) = 10 \).
   **Solution:** \( F(x) = 2x^2 + x^3 + 2x^4 + 2x + 3 \)
   Note: this problem is not a typo, solve it as written.

12. Find \( \int_{-10}^{6} |3x - 2| \, dx \)
   **Solution:** 640/3

13. \( \int_{0}^{5} \frac{1}{3} x^3 \, dx = \lim_{n \to \infty} R_n \), where \( R_n \) is the right hand Riemann sum. Find \( R_n \).
   **Solution:** \( R_n = \frac{625n^2 + 1250n + 625}{12n^2} \)
   One student proposed that the above is wrong and should instead be: \( R_n = \frac{625n^4 + 1250n^2 + 625}{12n^4} \) I haven’t had time to double check it.

14. \( \int_{1}^{2} 2x^2 + 1 \, dx = \lim_{n \to \infty} R_n \), where \( R_n \) is the right hand Riemann sum. Find \( R_n \).
   **Solution:** \( R_n = 3 + \frac{2(n+1)}{n} + \frac{(n+1)(2n+1)}{3n^2} \)

15. Let \( g(x) = x^3 \). Find the Riemann sum \( L_4 \) for \( g(x) \) on the interval \([1, 3]\).
   **Solution:** 14

16. Evaluate the following limit by first recognizing it as a Riemann sum for a function defined on \([0, 1]\)
   \[ \lim_{n \to \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}} \right) \]
   **Solution:** \( \int_{0}^{1} \sqrt{x} \, dx = 2/3 \)

17. Let \( g(x) = \int_{x}^{2} \tan t \, dt \). Find \( g'(x) \).
   **Solution:** \( g'(x) = 2x \tan(x^2) - \tan x \)

18. Let \( F(x) = \int_{\tan x}^{\sec x} \sqrt{t^2 + 3} \, dt \). Evaluate \( F'(0) \).
   **Solution:** \( F'(x) = \sec x \tan x \sqrt{\sec^2 x + 3} - \sec^2 x \sqrt{\tan^2 x + 3} \)
   \( F'(0) = -\sqrt{3} \)

19. If \( f(x) = \int_{0}^{x} (4 - t^2) e^t \, dt \), on what interval(s) is \( f \) decreasing and on what intervals is \( f \) increasing?
   **Solution:** Decreasing on \((-\infty, -2)\) and \((2, \infty)\), increasing on \((-2, 2)\).

20. Find all values of \( x \) where \( F(x) = \int_{0}^{x} \frac{t^3 - 3t^2 + 2t}{e^t} \, dt \) has a local maximum or local minimum.
   **Solution:** \( F'(x) = \frac{e^x - 3x^2 + 2x}{e^x} \)
   Solving \( F'(x) = 0 \) gives \( x = 0, 1, 2 \). Testing these points gives \( x = 0 \) is a min, \( x = 1 \) is a max, \( x = 2 \) is a min.

21. Find an antiderivative of \( e^{3x} \sin(3x^2 + \ln x) \).
   **Solution:** \( \int_{0}^{x} e^{3x} \sin(3x^2 + \ln x) \, dx \)

22. T/F. If \( f(x) \) is continuous and has a minimum of 3 on \([2, 4]\) then we can conclude \( \int_{2}^{4} f(x) \, dx \geq 6 \).
   **Solution:** True
23. T/F. Given \( \int_4^1 g(t) \, dt = -5 \) and \( \int_4^3 g(t) \, dt = 2 \) then \( \int_3^1 g(t) \, dt = -7 \).

Solution: False

24. Find \( \int_{\pi/4}^b \sin t \, dt \) for any \( b \).

Solution: \( \frac{1}{2} - \cos b \)

25. If \( f(5) = 10 \) and \( \int_5^{100} f'(x) \, dx = 73 \), what is \( f(100) \)?

Solution: \( f(100) - f(5) = 73 \) so \( f(100) = 83 \)

26. Find the value of \( t \) where \( f(t) \) has a local maximum:

\[
 f(t) = \int_0^t \frac{2x^2 + x - 10}{1 + \sin^2 x} \, dx.
\]

Solution: Solve \( f' = 0 \) to get \( t = 2 \) and \( t = -5/2 \). These these to find that \( f \) has a local max at \( t = -5/2 \) and a local min at \( t = 2 \).

27. Find an antiderivative of \( \frac{1}{\sqrt{x}} + e^{2x} + \sin 3x \).

Solution: \( 2\sqrt{x} + \frac{1}{2}e^{2x} - \frac{1}{4} \cos 3x + C \).

28. Suppose you know the following about a function \( f(x) \).

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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<tbody>
<tr>
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<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>-1</td>
<td>-1</td>
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</tbody>
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Find the Riemann Sum for \( \int_2^6 f(x) \, dx \) using 4 subintervals and right endpoints as sample points.

Solution: \( R_4 = 1(f(3) + f(4) + f(5) + f(6)) = 3 \)

29. If \( x^*_i \) is a sample point from the \( i \)th subinterval of a regular partition of \([1, 3]\) into \( n \) subintervals, and \( \Delta x \) is the length of each subinterval, find: \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{2} (x^*_i)^2 \Delta x \).

Solution: This is equal to \( \int_1^3 \frac{3}{2} x^2 \, dx = 13 \)

30. Let \( g(x) = \int_0^x f(t) \, dt \) where \( f \) is the function shown. Find \( g(5) \).

\[ g(5) \]

Solution: 5

31. Let \( h(x) = \int_1^{1/x} \sqrt{1 + u^3} \, du \). Find \( h'(1/2) \).

Solution: -12

32. Find \( \lim_{h \to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin \left( \frac{\pi t}{4} \right) \, dt \)

Solution:

\[
 \lim_{h \to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin \left( \frac{\pi t}{4} \right) \, dt = \frac{d}{dx} \left[ \int_2^x t^2 \sin \left( \frac{\pi t}{4} \right) \, dt \right]_{x=2} = 4 \sin \left( \frac{\pi}{2} \right) \, dt = 4
\]