Math 132 - September 11, 2017 (Exam 1 Review)

Solutions

1. Suppose $\int_1^3 f(x)\,dx = 6$ and $\int_1^3 g(x)\,dx = 2$. What is $\int_1^3 (2f(x) - 3g(x))\,dx$?

   Solution: 6

2. Suppose $\int_1^4 f(x) = 8$, $\int_1^2 f(x) = 4$, $\int_3^4 f(x) = 2$, what is $\int_2^3 f(x)$?

   Solution: 2

3. Find $\int_0^1 x\left(\sqrt{x} + \sqrt[3]{x}\right)\,dx$

   Solution: 29/35

4. If $g(x) = \int_1^{\sqrt{x}} \sin(t^2)\,dt$, find $g'(x)$.

   Solution: $\dfrac{\sin x}{2\sqrt{x}}$

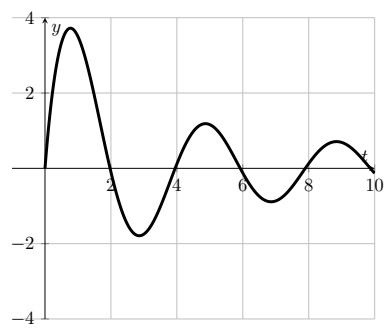
5. Let $g(x) = \int_0^x f(t)\,dt$ where $f(t)$ is the graph below. Determine which of the statements are true:

   (a) g attains an absolute maximum at $x = 2$ **Correct**

   (b) g has a local maximum at $x = 5$

   (c) g has a local minimum at $x = 4$ **Correct**

   (d) g is concave down on $[0, 2]$



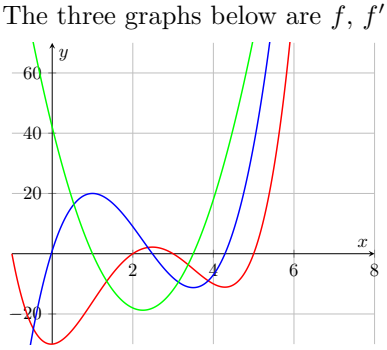
6. Suppose $f''(x) = -9\sin 3x$ and $f'(0) = 0$ and $f(0) = 2$. Find $f(\pi/4)$.

   Solution: $-3\pi/4 + 1/\sqrt{2} + 2$

7. If $f'''(x) = \sin x$, $f(0) = -3$, $f'(0) = 4$ and $f''(0) = 1$. What is $f(x)$?

   Solution: $\cos x + x^2 + 4x - 4$

8. The three graphs below are f, f' and f''. Identify which is which.



   Solution: Red is f, blue is f' and green is f''

9. Write $\int_2^{10} x^6\,dx$ as a limit of Riemann Sums (right handed sums). (Your answer should be in summation notation.)

   Solution: $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{8i}{n}\right)^6 \cdot \frac{8}{n}$

10. Suppose you know that $\displaystyle\int_0^b f(x)\,dx = \ln(b+1)$ for $b > 0$. What is $\displaystyle\int_3^5 \big(3f(x) - 2\big)\,dx$?

    Solution: $3\ln(3/2) - 4$

    Note: this was changed from $\int_2^b f(x)\,dx$ to $\int_0^b f(x)\,dx$. Without this change there is a clear issue with $b = 2$ (which should give an integral of 0, not $\ln 3$.

11. Find a function $F(x)$ such that $F''(x) = 4 + 6x + 24x^2$, $F(0) = 3$, $F(1) = 10$.
Solution: $F(x) = 2x^2 + x^3 + 2x^4 + 2x + 3$
Note: this problem is not a typo, solve it as written.

12. Find $\int_{-10}^{6} |3x - 2|\, dx$
Solution: $640/3$

13. $\displaystyle\int_0^5 \frac{1}{3}x^3\, dx = \lim_{n\to\infty} R_n$, where R_n is the right hand Riemann sum. Find R_n.
Solution: $R_n = \dfrac{625n^2 + 1250n + 625}{12n^2}$
One student proposed that the above is wrong and should instead be: $R_n = \dfrac{625n^4 + 1250n^2 + 625}{12n^4}$ I haven't had time to double check it.

14. $\displaystyle\int_1^2 2x^2 + 1\, dx = \lim_{n\to\infty} R_n$, where R_n is the right hand Riemann sum. Find R_n.
Solution:
$$R_n = 3 + \frac{2(n+1)}{n} + \frac{(n+1)(2n+1)}{3n^2}$$

15. Let $g(x) = x^3$. Find the Riemann sum L_4 for $g(x)$ on the interval $[1, 3]$.
Solution: 14

16. Evaluate the following limit by first recognizing it as a Riemann sum for a function defined on $[0, 1]$
$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$$
Solution: $\displaystyle\int_0^1 \sqrt{x}\, dx = 2/3$

17. Let $g(x) = \displaystyle\int_x^{x^2} \tan t\, dt$. Find $g'(x)$.
Solution: $g'(x) = 2x\tan(x^2) - \tan x$

18. Let $F(x) = \displaystyle\int_{\tan x}^{\sec x} \sqrt{t^2 + 3}\, dx$. Evaluate $F'(0)$.
Solution: $F'(x) = \sec x \tan x\sqrt{\sec^2 x + 3} - \sec^2 x\sqrt{\tan^2 x + 3}$.
$F'(x) = -\sqrt{3}$.

19. If $f(x) = \displaystyle\int_0^x (4 - t^2)e^{t^3}\, dt$, on what interval(s) is f decreasing and on what intervals is f increasing?
Solution: Decreasing on $(-\infty, -2)$ and $(2, \infty)$, increasing on $(-2, 2)$.

20. Find all values of x where $F(x) = \displaystyle\int_0^x \frac{t^3 - 3t^2 + 2t}{e^t}\, dt$ has a local maximum or local minimum.
Solution: $F'(x) = \frac{x^3 - 3x^2 + 2x}{e^x}$
Solving $F'(x) = 0$ gives $x = 0, 1, 2$. Testing these points gives $x = 0$ is a min, $x = 1$ is a max, $x = 2$ is a min.

21. Find an antiderivative of $e^{x^2}\sin(3x^2 + \ln x)$.
Solution: $\displaystyle\int_0^x e^{x^2}\sin(3x^2 + \ln x)\, dx$

22. T/F. If $f(x)$ is continuous and has a minimum of 3 on $[2, 4]$ then we can conclude $\int_2^4 f(x)\, dx \geq 6$.
Solution: True

23. T/F. Given $\int_1^4 g(t)\,dt = -5$ and $\int_4^3 g(t)\,dt = 2$ then $\int_3^1 g(t)\,dt = -7$.

Solution: False

24. Find $\int_{\pi/3}^{b} \sin t\,dt$ for any b.

Solution: $\frac{1}{2} - \cos b$

25. If $f(5) = 10$ and $\int_5^{100} f'(x)\,dx = 73$, what is $f(100)$?

Solution: $f(100) - f(5) = 73$ so $f(100) = 83$

26. Find the value of t where $f(t)$ has a local maximum:
$f(t) = \displaystyle\int_0^t \frac{2x^2 + x - 10}{1 + \sin^2 x}\,dx.$

Solution: Solve $f' = 0$ to get $t = 2$ and $t = -5/2$. These these to find that f has a local max at $t = -5/2$ and a local min at $t = 2$.

27. Find an antiderivative of $\frac{1}{\sqrt{x}} + e^{2x} + \sin 3x$.

Solution: $2\sqrt{x} + \frac{1}{2}e^{2x} - \frac{1}{3}\cos 3x + C$.

28. Suppose you know the following about a function $f(x)$.

x	0	1	2	3	4	5	6	7	8
$f(x)$	-1	3	1	2	2	1	-1	-1	1

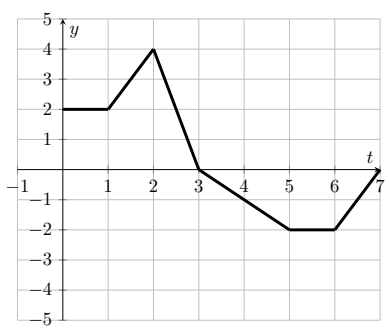
Find the Riemann Sum for $\int_2^6 f(x)\,dx$ using 4 subintervals and right endpoints as sample points.

Solution: $R_4 = 1(f(3) + f(4) + f(5) + f(6)) = 3$

29. If x_i^* is a sample point from the ith subinterval of a regular partition of $[1, 3]$ into n subintervals, and Δx is the length of each subinterval, find: $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{2}(x_i^*)^2 \Delta x.$

Solution: This is equal to $\int_1^3 \frac{3}{2}x^2\,dx = 13$

30. Let $g(x) = \int_0^x f(t)\,dt$ where f is the function shown. Find $g(5)$.



Solution: 5

31. Let $h(x) = \displaystyle\int_1^{1/x} \sqrt{1 + u^3}\,du$. Find $h'(1/2)$.

Solution: -12

32. Find $\displaystyle\lim_{h\to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt$

Solution:

$$\lim_{h\to 0} \frac{1}{h} \int_2^{2+h} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt = \frac{d}{dx}\bigg|_{x=2} \int_2^{x} t^2 \sin\left(\frac{\pi t}{4}\right)\,dt = x^2 \sin\left(\frac{\pi x}{4}\right)\,dt\bigg|_{x=2} = 4\sin\left(\frac{\pi}{2}\right)\,dt = 4$$