Math 132: Discussion Session: Week 2

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn this paper in at the end of class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Scoring:

<table>
<thead>
<tr>
<th>Correct answers</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>0%</td>
</tr>
<tr>
<td>4–6</td>
<td>80%</td>
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<tr>
<td>7–14</td>
<td>100%</td>
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</tbody>
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Group Members:

5.1–5.2: Riemann Sums and Limits of Riemann Sums.

(1) \( L_3 = \)

(2) \( M_5 = \)

(3) (a) \( R_6 = \)

(b) \( R_6 = \sum_{i=1}^{6} \)

(c) \( R_{127} = \sum_{i=1}^{127} \)

(d) \( R_{127} = \)

(e) \( R_n = \sum_{i=1}^{n} \)

(f) \( R_n = \)

(g) \( \int_{2}^{5} (5x^2 - 3) \, dx = \)

(4) (a) \( R_n = \sum_{i=1}^{n} \)

(b) \( R_n = \)

(c) \( \int_{0}^{2} (x^3 - 2x) \, dx = \)

5.2: Definite Integrals as Signed Area

(1) \( \int_{0}^{2} \sqrt{16 - 4x^2} \, dx = \)

(2) \( \int_{-2}^{5} \left( 4 + x - 2|x| \right) \, dx = \)
5.1–5.2: Riemann Sums and Limits of Riemann Sums. Useful formulas:

\[ \sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}. \]

1. Let \( f(x) = \sin^2(x) \). Compute the Riemann sum \( L_3 \) with three intervals, sampling at left endpoints, on \( \left[ \frac{\pi}{6}, \frac{5\pi}{12} \right] \).

2. Let \( f(x) = \ln(2x) \). Compute the Riemann sum \( M_5 \) with five intervals, sampling at midpoints, on \( [1, 3] \).

3. Let \( f(x) = 5x^2 - 3 \) on \( [2, 5] \).
   
   (a) Compute the Riemann sum \( R_6 \) with six intervals, sampling at right endpoints.
   
   (b) Write down an expression for \( R_6 \) using \( \sum_{i=1}^{n} \) notation. The only variable in your final answer should be \( i \).
   
   (c) Write down an expression for \( R_{127} \) using \( \sum_{i=1}^{127} \) notation. The only variable in your final answer should be \( i \).
   
   (d) Use the formulas at the top of the page to write down an expression for \( R_{127} \). You don’t need to simplify, but there should be no variables or summations in your final answer.
   
   (e) Write down an expression for \( R_n \) using \( \sum_{i=1}^{n} \) notation. The only variables in your final answer should be \( i \) and \( n \).
   
   (f) Use the formulas at the top of the page to write down an expression for \( R_n \). There should be no summations in your final answer, and the only variable in your answer should be \( n \).
   
   (g) Compute the definite integral \( \int_{2}^{5} (5x^2 - 3) \, dx \) by computing \( \lim_{n \to \infty} R_n \).
   
   (h) Compute the definite integral \( \int_{2}^{5} (5x^2 - 3) \, dx \) by computing the antiderivative of \( 5x^2 - 3 \).

4. Let \( f(x) = x^3 - 2x \) on \( [0, 2] \).
   
   (a) Write down an expression for \( R_n \) using \( \sum_{i=1}^{n} \) notation. The only variables in your final answer should be \( i \) and \( n \).
   
   (b) Use the formulas at the top of the page to write down an expression for \( R_n \). There should be no summations in your final answer, and the only variable in your answer should be \( n \).
   
   (c) Compute the definite integral \( \int_{0}^{2} (x^3 - 2x) \, dx \) by computing \( \lim_{n \to \infty} R_n \).
   
   (d) Compute the definite integral \( \int_{0}^{2} (x^3 - 2x) \, dx \) by computing the antiderivative of \( x^3 - 2x \).

5.2: Definite Integrals as Signed Area. Compute the following definite integrals by interpreting them as areas

1. \( \int_{0}^{2} \sqrt{16 - 4x^2} \, dx \)

2. \( \int_{-2}^{5} \left(4 + x - 2 |x|\right) \, dx \). Hint: Consider \( x < 0 \) and \( x > 0 \) separately to draw the graph of this function.