Math 132 - August 28, 2017

Solutions

4.9: Anti-derivatives, 5.1: Areas/Distances

- Antiderivatives: $\int f(x)\,dx$
- Summation notation: $\sum_{i=1}^{10} a_i$
- Area under a graph of a function
- Left hand sums, right hand sums, mid-point sums, trapezoid sums

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Suppose there is a function $f(x)$ and you know that $f'(x) = 6x^2 - 2x$.
   What is $f(x)$?

   (a) $f(x) = 12x - 2$

   (b) $f(x) = 2x^3 - x^2 + 7$

   (c) $f(x) = 2x^3 - x^2 + C$

   (d) None of the above

   (e) Unable to determine $\boxed{\textbf{Correct}}$

Lecture Notes:

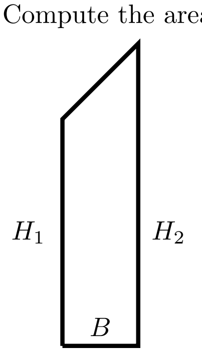
- What is an antiderivative?
- What is the *most general* antiderivative?
- How do you check to see if you found an antiderivative?
- We write "the most general antiderivative" using integral sign:

$$\int 6x^2 - 2x\,dx = 2x^2 - x^2 + C$$

2. Compute

$$\sum_{i=2}^{6} i^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$$

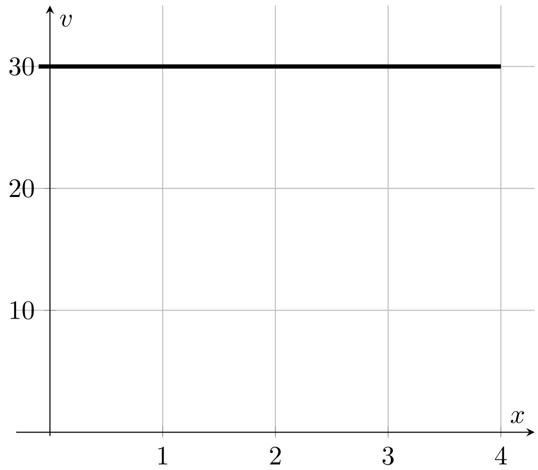
3. Compute the area of the trapezoid:



Solution: Break it up into rectangles and triangles: $A = \frac{1}{2}(H_1 + H_2)B$

4. If you travel at a constant speed of 10 miles per hour, how far do you travel in 3 hours?

   Solution: 30 miles, of course. But the interesting thing here is looking at areas. If we plot the velocity function $v(t) = 30$ and find the area between $t = 0$ to $t = 3$, then this is distance traveled.



Class Problems

Lecture Notes:

- Our goal is going to be to compute areas "under" functions. The strategy is to going to be to take limits of Riemann sums.

  Approximate $\int_0^1 x^2\,dx$ by finding:

$$\begin{aligned} LHS_4 &= 7/32 \approx .21875 \\ RHS_4 &= 15/32 \approx .46875 \\ M_4 &= 11/32 \approx .34375 \\ T_4 &= 21/64 \approx .328125 \\ \text{exact} &= 1/3 \end{aligned}$$

5. **Clicker** For $\displaystyle\int_2^4 2x + 3\,dx$ compute LHS_4.

   (a) 17 **Correct**
   (b) 18
   (c) 19
   (d) π
   (e) I have no idea

6. For $\displaystyle\int_2^4 2x + 3\,dx$ compute RHS_4, T_4 and M_4.

   Solution: $LHS_4 = 17$, $RHS_4 = 19$, $T_4 = 18$, $M_4 = 18$

7. **Clicker** If you approximate $\int_1^9 28 - x^3\,dx$, using left hand sums, right hand sums, midpoint sums and trapezoid sums. Which of the following is the largest:

   (a) LHS_{32} **Correct**
   (b) RHS_{32}
   (c) M_{32}
   (d) T_{32}
   (e) None of the above

8. **Clicker** If you approximate $\int_1^9 x^3 - 28\,dx$, using left hand sums, right hand sums, midpoint sums and trapezoid sums. Which of the following is the largest:

   (a) LHS_{32}

(b) RHS_{32} $\boxed{\textbf{Correct}}$

(c) T_{32}

(d) M_{32}

(e) None of the above

9. $\boxed{\textbf{Clicker}}$ If you approximate $\int_0^{3\pi/4} \sin x \, dx$, using left hand sums, right hand sums, midpoint sums and trapezoid sums. Which of the following is the largest:

   (a) LHS_{32}

   (b) RHS_{32} $\boxed{\textbf{Correct}}$

   (c) T_{32}

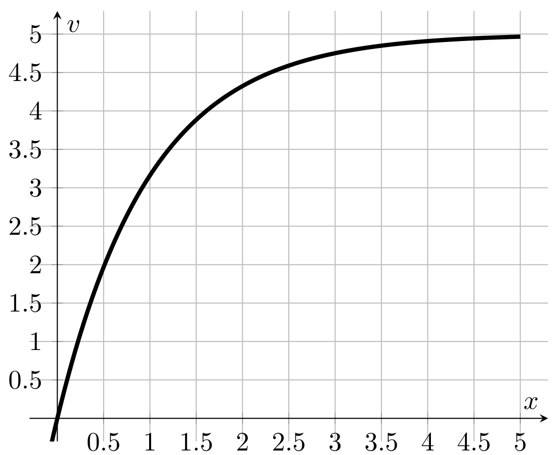
   (d) M_{32}

   (e) None of the above

   Solution: Note: questions like this are not obvious and can be more or less impossible to figure out without actually computing it!

Lecture Notes: Computing areas is actually useful. If you plot velocity then the area under the velocity curve is the distance travelled. See why this works for constant velocity.

10. Given the graph of velocity below, estimate the distance traveled from time $t = 1$ to $t = 4$.



   Solution: Units are important and they are ignored, but answer is distance is approximately 18.

Lecture Notes: How to compute actual area:

(1) Draw the domain of interest ($x = a$ to $x = b$)

(2) Let $\Delta x = \frac{b-a}{n}$.

(3) Chop the domain up into n sub intervals. $x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \ldots x_n = a + n\Delta x = b$

(4) Compute a left or right hand sum (just do a right hand sum):

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

(5) Use math to get a closed form expression for R_n

(6) Take the limit: Area $= \lim_{n\to\infty} R_n$