Math 132 - December 9, 2016

Solutions

Final Exam: Sections 5.1-11.10

- Final exam: Fri Dec 16 10:30AM
- Rough final exam breakdown:
  Exam 1: 20%, Exam 2: 20%,
  Exam 3: 20%, Post Exam 3: 40%.
- Reading Week:
  - Calculus Help Room Open
  - RPM Help
  - Thornton/Bickel Office Hours
  - Review Session (see emails)

1. Find $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx$.
   Solution: $u = \tan x$ gives answer $1/4$.

2. Find $\int xe^x \, dx$
   Solution:

3. Find $\int \frac{3}{x^2+x-2} \, dx$
   Solution: $\ln|x-1| - \ln|x+2| + C$

4. Find the area between curves $y = 2 - x^2$ and $y = 2 - 2x$.
   Solution: $4/3$

5. Find the volume obtained by rotating the region between the y axis and the curve $x = 2/y$ for $1 \le y \le 4$, about the y-axis.
   Solution: $V = \pi \int_1^4 (4/y^2) \, dy = 3\pi$

6. Find $\int_0^\infty \frac{x}{(x^2+2)^2} \, dx$
   Solution: $1/4$

7. Find the sum $\sum_{n=1}^\infty \frac{2^n-1}{3^n}$
   Solution: $3/2$

8. Find the interval of convergence of $\sum_{n=1}^\infty \frac{2^n(x-4)^n}{n3^n}$
   Solution: $[5/2, 11/2)$.

9. The Maclaurin series for $x\cos(x/2) = \sum_{n=0}^\infty a_n x^n$. Find a_5.
   Solution: $1/384$.

10. If $\int \arctan(x^2) \, dx = C + \sum_{n=1}^\infty a_n x^n$, find a_7.
    Solution: $-1/21$.

11. If $\sin x = \sum_{n=0}^\infty a_n (x - \pi/3)^n$. Find a_3.
    Solution: $-1/12$.

12. How man terms of the series are needed to approximate $\int_0^1 \cos(x^3) \, dx$ to an error less than 10^{-4}? What is this estimate?
    Solution: 3 terms needed: $1 - 1/14 + 1/312 = .931777$.

13. Find $\int_1^e x^2 \ln x \, dx$
    Solution: $(2e^3 + 1)/9$

14. Find $\int -\frac{1}{x^2-x} \, dx$
    Solution: $\ln|x/(x-1)| + C$

15. Find the length of $x = (2/3)y^{3/2}$, $0 \le y \le 3$.
    Solution: $14/3$.

16. Find T_2, centered at $x = 1$ for $f(x) = 1/x$.
    Solution: $T_2(x) = 1 - (x-1) + (x-1)^2$.

17. if $xe^{2x} = \sum_{n=1}^{\infty} b_n x^n$, find b_4.
Solution: $4/3$.

18. Find Maclaurin series for $x^2 - x \arctan x$
Solution: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+3}$.

19. Find $\int_0^8 \frac{1}{\sqrt[3]{8-x}}\, dx$
Solution: 6

20. Find the sum $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \frac{48}{625} - \cdots$
Solution: $15/7$

21. Find a function represented by Maclaurin series:
$1 + 3x + 9x^2 + 27x^3 + 81x^4 + \cdots$
Solution: $1/(1-3x)$.

22. Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(3x-2)^n}{2^n}$
Solution: $(0, 4/3)$

23. Find $\int_1^2 (2x-1)\cos(x^2 - x)\, dx$
Solution: $\sin 2$

24. Find the radius of convergence of $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$
Solution: 4.

25. If $\frac{1}{x^2} = \sum_{n=0}^{\infty} d_n (x-2)^n$ find d_3.
Solution: $-24/192$

26. Find $f(x)$ and b if: $3 + \displaystyle\int_b^x \frac{f(t)}{t^4}\, dt = 24x^{-3}$..
Solution: Plugging in $x = b$ makes the integral equal to 0 and gives the equation $3 + 0 = 24b^{-3}$. Solving gives $b = 2$. Take the derivative of both sides to find $f(x)$: $\frac{f(x)}{x^4} = -72x^{-4}$, which gives $f(x) = -72$.

27. Let $f(x) = \int_x^2 \frac{5+t^4}{\sqrt{1+t^3}}\, dt$. Find $f'(2)$.
Solution: -7.

28. Determine convergence or divergence of $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)^n$
Solution: Diverges since $\lim a_n \neq 0$.

29. Determine convergence or divergence of $\sum_{n=0}^{\infty} \frac{(-1)^n n^{2/3}}{1+n^{4/3}}$.
Solution: Converges conditionally but not absolutely.

30. Determine convergence or divergence of $\sum_{n=1}^{\infty} \left(\frac{3+n}{3n}\right)^n$
Solution: Converges absoutely by root test.

31. Let $f(x) = \frac{1}{72} x^3 e^{2x^2}$. Find $f^{(7)}(0)$.
Solution: Write out the Maclaurin series and look at the 7th term to get answer: 140.

32. Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{4^n \sqrt{n+1}}$
Solution: Radius of convergence is 4. Interval: $(-7, 1]$

33. Let T_2 be the degree 2 taylor polynomial centered at 2 for $\ln x$. Find $T_2(3) - \ln 2$.
Solution: $3/8$.

34. Find the coefficient of x^5 in the Maclaurin series for $\frac{8x}{4-x^2}$.
Solution: $1/8$

35. Find the coefficient of x^4 in the Maclaurin series for $\frac{1}{(1+x^2)^{1/3}}$.
Solution: $2/9$

36. Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{n}{b^n}(x-1)^n$ where $b > 0$.
Solution: $R = b$ and $I = (1-b, 1+b)$.

37. Find all p such that $\int_2^{\infty} \frac{(\ln x)^p}{x}\, dx$ converges.
Solution: $p < -1$.

38. Find the area between $y = 10/x$ and $y = 7 - x$.
Solution: Area is $\left[7x - \frac{x^2}{x} - 10\ln x\right]_2^5$ (whatever that equals).

39. Find the sum of $1 - e + e^2/2! - e^3/3! + e^4/4! - \cdots$
Solution: $1/e^e$.

40. If $g(x) = \int_{x^2}^3 \sqrt{1+t^3}\, dt$ then what is $g'(1)$?
Solution: $-2\sqrt{2}$

41. (T/F) If $r > 1$ and $a \neq 0$ then $\sum_{n=1}^{\infty} ar^n$ converges.
Solution: False

42. (T/F) If $\sum_{n=1}^{\infty} c_n(-2)^n$ converges then $\sum_{n=1}^{\infty} \frac{c_n}{2^n}$ converges.
Solution: True

43. (T/F) If $\sum_{n=1}^{\infty} c_n(x-10)^n$ converges for all x then $\sum_{n=1}^{\infty} c_n n(x-10)^{n-1}$ converges for all x. converges.
Solution: True.

44. (T/F) If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.
Solution: False: maybe it does maybe it doesn't (see if you can find examples of both).

45. (T/F) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2\pi}{n} \sin\left(\frac{2\pi i}{n}\right) = 0$
Solution: True. Write it as a Riemann sum.

46. Approximate $\int_0^3 x^2\, dx$ by using the midpoint rule and 3 subdivisions.
Solution: $35/4$.

47. Let $g(x) = \int_2^{1/x} \arctan t\, dt$. Find $g'(x)$.
Solution: $(-1/x^2)\arctan(1/x)$.

48. Find $\int \frac{x+1}{x^2+1}\, dx$
Solution: $\frac{1}{2}\ln|x^2+1| + \arctan x + C$

49. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = x^3$, $y = 8$ and $x = 0$.
Solution: $96\pi/5$

50. Given that $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$. How many terms must be added to obtain a partial sum within $\frac{1}{100}$ of $\ln 2$?
Solution: If you add 99 terms then the next term is $-1/100$ and thus the maximum error is $1/100$. Thus, you need 99 terms.