# Math 132: Discussion Session: Week 15

**Directions:** In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1 and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to the other questions.

**Additional Instructions:** It is okay if you do not completely finish all of the problems, but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

## Group Members

## Group Answer and Work

1. Find the Taylor series for $f(x) = x^5 + 2x^3 + x$, centered at 2. What is the interval of convergence of the Taylor series?

$f(x) = x^5 + 2x^3 + x \qquad f(2) = 50$

$f'(x) = 5x^4 + 6x^2 + 1 \qquad f'(2) = 150$

$f''(x) = 20x^3 + 12x \qquad f''(2) = 184$

$f^{(3)}(x) = 60x^2 + 12 \qquad f^{(3)}(2) = 252$

$f^{(4)}(x) = 120x \qquad f^{(4)}(2) = 240$

$f^{(5)}(x) = 120 \qquad f^{(5)}(2) = 120$

$f^{(6)}(x) = 0$

$\vdots$ all rest are zero

$$\text{Taylor Series} = 50 + 150(x-2) + \frac{184}{2!}(x-2)^2 + \frac{252}{3!}(x-2)^3 + \frac{240}{4!}(x-2)^4 + \frac{120}{5!}(x-2)^5$$

$$= 50 + 150(x-2) + 92(x-2)^2 + 42(x-2)^3 + 10(x-2)^4 + (x-2)^5$$

B/c it's a polynomial, the interval of convergence is trivially $(-\infty, \infty)$.

Solutions

# Math 132 Discussion Session: Week 15

## 11.10: Taylor and Maclaurin series

1. Find the Taylor series for $f(x) = x^5 + 2x^3 + x$, centered at 2. What is the interval of convergence of the Taylor series? $50 + 105(x-2) + 92(x-2)^2 + 42(x-2)^3 + 10(x-2)^4 + (x-2)^5$

$R = \infty$

$(-\infty, \infty)$

2. Find the first four nonzero terms of the Taylor series for $f(x)$ centered at the given value.

a. $f(x) = \ln x$ centered at 1 $\quad (x-1) - 1/2(x-1)^2 + 1/3(x-1)^3 - 1/4(x-1)^4$

b. $f(x) = x^{\frac{1}{3}}$ centered at 8 $\quad 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20,736}(x-8)^3$

c. $f(x) = \sin x$ centered at $\frac{\pi}{6}$ $\quad \frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/6) - 1/4(x-\pi/6)^2 - \sqrt{3}/12(x-\pi/6)^3$

d. $f(x) = e^x \cos x$ centered at 0 (Hint: Multiply the Maclaurin series for $e^x$ and $\cos x$ together.)

$= [1 + x + x^2/2 + x^3/6 + \frac{x^4}{24} + \ldots][1 - x^2/2 + x^4/24 + \ldots] = 1 + x - x^3/3 - x^4/6 + \ldots$

2. Find the Maclaurin series for $f(x)$.

a. $f(x) = \frac{2x^2}{(1-4x)^2} = 2x^2 \sum_{n=1}^{\infty} 4^{n-1} \cdot n \cdot x^{n-1} = \sum_{n=1}^{\infty} 2 \cdot 4^{n-1} \cdot n \cdot x^{n+1}$

b. $f(x) = \frac{1}{(2-x)^3}$ $\quad \sum_{n=2}^{\infty} \frac{(n-1)n\,x^{n-2}}{2^{n+1}}$

c. $f(x) = 2^x$ $\quad \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n$

d. $f(x) = x\cos(2x)$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+1}}{(2n)!}$

3. Find the Taylor series for $f(x)$ with the given center.

a. $f(x) = \ln x$ centered at 2 $\quad \ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n 2^n} (x-2)^n$

b. $f(x) = \sin(2x)$ centered at $\pi$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (x-\pi)^{2n+1}}{(2n+1)!}$

c. $f(x) = x^2 \ln(1 + x^3)$ centered at 0 $\quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+5}}{n+1}$

4. Evaluate the indefinite integral as an infinite series:

a. $\int \sin(x^3)\, dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(2n+1)!(6n+4)}$

b. $\int \frac{\cos x - 1}{x}\, dx = C + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2n(2n)!}$