Solution

Math 132: Discussion Session: Week 14

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1 and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to the other questions.

Additional Instructions: It is okay if you do not completely finish all of the problems, but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answer and Work

1. Determine the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n \sqrt{n^2+1}}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{2^{n+1} \cdot \sqrt{(n+1)^2+1}} \cdot \frac{2^n \cdot \sqrt{n^2+1}}{x^n} \right| = \lim_{n\to\infty} \frac{|x|}{2} \cdot \sqrt{\frac{n^2+1}{(n+1)^2+1}} = \frac{|x|}{2} < 1$$

conv if $|x| < 2 \Rightarrow -2 < x < 2$ $\boxed{\text{radius of convergence} = 2}$

div if $|x| > 2$

check endpoints

$x=-2$ $\sum_{n=0}^{\infty} \frac{(-1)^n(-2)^n}{2^n\sqrt{n^2+1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ compare to $\sum_{n=0}^{\infty} \frac{1}{n}$ div b/c p-series w/ $p=1$

$\lim_{n\to\infty} \frac{1/\sqrt{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$ so $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ also diverges by the Limit Comparison Test

$x=2$ $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{2^n\sqrt{n^2+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$ Alternating series Test

$b_n = \frac{1}{\sqrt{n^2+1}} > 0$, $b_{n+1} = \frac{1}{\sqrt{(n+1)^2+1}} \le \frac{1}{\sqrt{n^2+1}} = b_n$, $\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = 0$.

$\therefore$ series converges by the Alternating Series Test

$\boxed{\text{interval of convergence: } -2 < x \le 2 \text{ or } (-2, 2]}$

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1. Determine the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n \sqrt{n^2+1}}$

11.8: Power Series

2. Find the interval of convergence of the following power series

a. $\sum_{n=1}^{\infty} n x^n$ $\quad (-1, 1)$

b. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^3} x^n$ $\quad (-\infty, \infty)$

c. $\sum_{n=1}^{\infty} \frac{x^n}{\ln n}$ $\quad [-1, 1)$

d. $\sum_{n=1}^{\infty} \frac{2^n}{3n}(x+3)^n$ $\quad [-7/2, 5/2)$

e. $\sum_{n=1}^{\infty} \frac{(-5)^n}{n!}(x+10)^n$ $\quad (-\infty, \infty)$

f. $\sum_{n=1}^{\infty} e^n (x-2)^n$ $\quad (2 - 1/e, 2 + 1/e)$

11.9: Representing Functions as Power Series

3. Find a power series representation for the following functions.

a. $f(x) = \frac{x-1}{x+2}$ $\quad -\frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n 3 x^n}{2^{n+1}}$

b. $f(x) = \frac{2x-4}{x^2-4x+3}$ (Hint: First use partial fractions.) $\quad \sum_{n=0}^{\infty} \left[-1 - \frac{1}{3^{n+1}}\right] x^n$

c. $f(x) = \frac{x}{(1+4x)^2}$ $\quad \sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}$

d. $f(x) = \ln(1+x^4)$ $\quad \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{4n}}{n}$

4. Evaluate the following indefinite integrals as power series.

a. $\int \cancel{\frac{x}{1+x^3}\,dx}$ $\frac{t}{1-t^8}\,dt$ $\quad C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}$

b. $\int x^2 \ln(1+x)\,dx$ $\quad C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+3}}{n(n+3)}$