Solutions

Math 132: Discussion Session: Week 12

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1 and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to the other questions.

Additional Instructions: It is okay if you do not completely finish all of the problems, but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answer and Work

1. a. Determine whether $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+\ln n}}$ converges or diverges. State any tests used and show that all conditions of the test are satisfied.

Note that $\frac{1}{n\sqrt{n+\ln n}} < \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$ & $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges b/c it's a p-series w/ $p = 3/2 > 1$. Thus, the series $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+\ln n}}$ also converges by the Comparison Test

b. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ converge absolutely, converges conditionally, or diverges. State any tests used and show that all conditions of the test are satisfied.

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ Limit compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, diverges b/c p-series with $p = 1/2$.

Since $\lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} \Big/ \frac{1}{\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}} = \frac{1}{2}$,

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ also diverges by the Limit Comparison Test.

However, $\frac{1}{\sqrt{n}+\sqrt{n+1}} > 0$, $\frac{1}{\sqrt{n+1}+\sqrt{n+2}} < \frac{1}{\sqrt{n}+\sqrt{n+1}}$, & $\lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} = 0$

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ converges by the Alternating Series Test and so, conditionally converges.

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1. a. Determine whether $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+\ln n}}$ converges or diverges. State any tests used and show that all conditions of the test are satisfied.

   b. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ converge absolutely, converges conditionally, or diverges. State any tests used and show that all conditions of the test are satisfied.

11.4: Comparison Tests

2. Determine whether the following series converge or diverge using any of the methods discussed so far in class.

   a. $\sum_{n=1}^{\infty} \frac{\cos n}{n^{\frac{3}{2}}}$ C

   b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ C

   c. $\sum_{n=1}^{\infty} ne^{-.03n}$ C

   d. $\sum_{n=1}^{\infty} (\ln(n+1) - \ln n)$ D

   e. $\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi}{n}\right)$ C

   f. $\sum_{n=1}^{\infty} \frac{n}{3^n - 2^n}$ C

11.5/11.6: Alternating Series and Absolute Convergence

3. Determine whether the following series converge absolutely, converge conditionally, or diverge.

   a. $\sum_{n=1}^{\infty} (-1)^n (\ln(n+1) - \ln n)$ Conv. Conditionally

   b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.01}\ln(n+1)}$ Conv. Absolutely

   c. $\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{(n!)^2}$ Diverges

d. $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{1}{n}\right)$ converges conditionally

4. Assume $\sum_{n=1}^{\infty} a_n$ is an absolutely convergent series. Determine whether the following series are convergent or divergent.

a. $\sum_{n=1}^{\infty}\left(a_n + \frac{1}{n^2}\right)$ C

b. $\sum_{n=1}^{\infty}(-1)^n a_n$ C

c. $\sum_{n=1}^{\infty}\frac{1}{1+a_n^2}$ D

d. $\sum_{n=1}^{\infty}\frac{|a_n|}{n}$ C